An introduction to Psychometric Theory with applications in R
Structural Equation Modeling and applied scale construction

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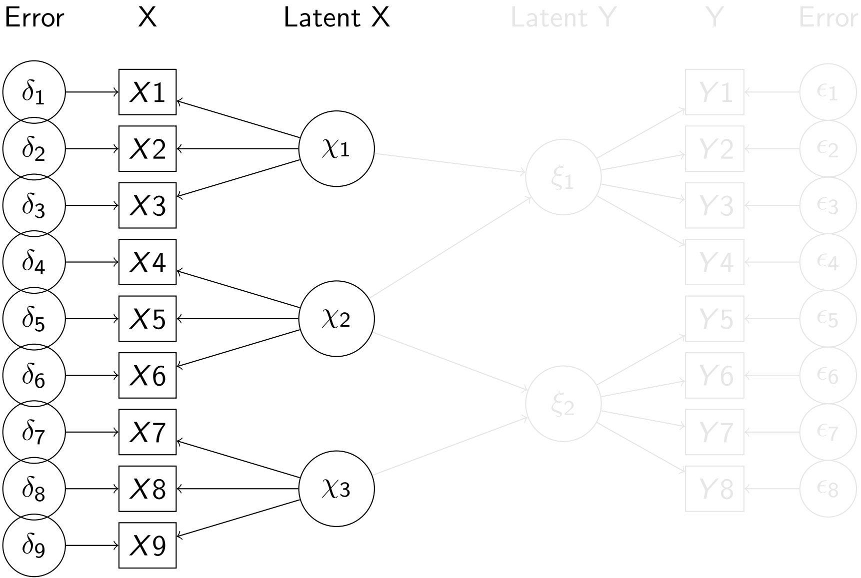
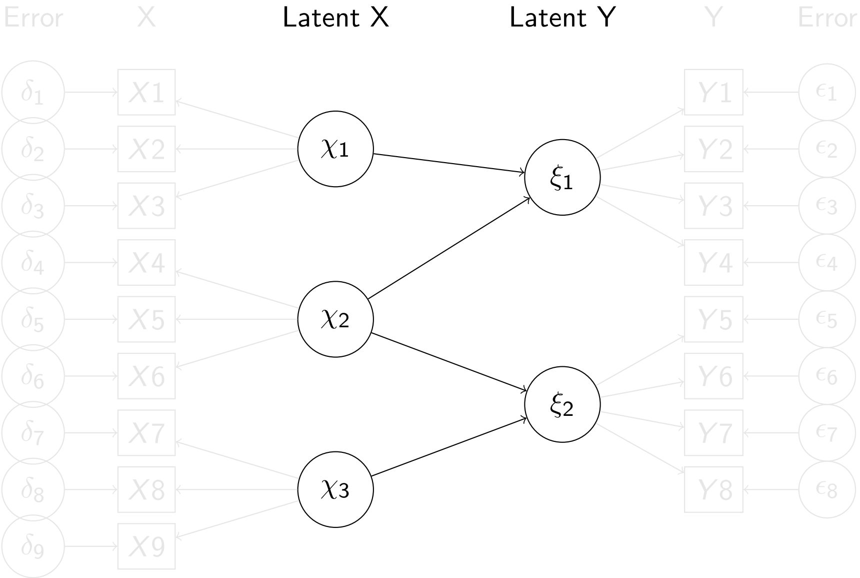
Outline

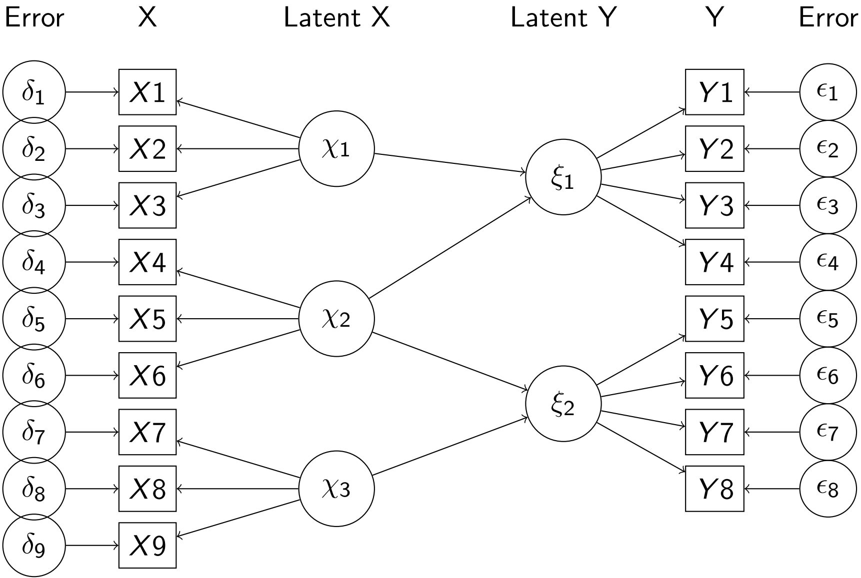
Measurement: A latent variable approach.



Theory


Error
X
Latent X
Latent Y
Y
Error
δ1
δ2
δ3
δ4
δ5
δ6
δ7
δ8
δ9
X1
X2
X3
X4
X5
X6
X7
X8
X9
χ1
χ2
χ3
ξ1
ξ2
Y1
Y2
Y3
Y4
Y5
Y6
Y7
Y8
ϵ1
ϵ2
ϵ3
ϵ4
ϵ5
ϵ6
ϵ7
ϵ8

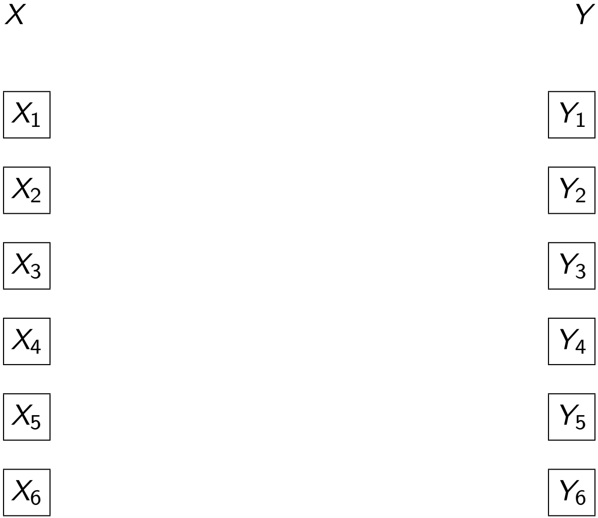
Psychometric Theory: A conceptual Syllabus



Two types of variables, three types of relationships

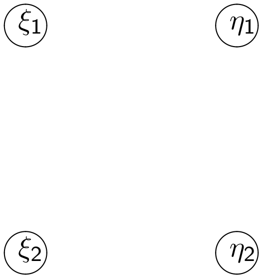
1. Variables
   1. Observed Variables (X, Y)
   2. Latent Variables (ξ η ϵ ζ)
2. Three kinds of variance/covariances
   1. Observed with Observed C_{xy} or σ_{xy}
   2. Observed with Latent λ
   3. Latent with Latent ϕ
3. Direction
   - Bidirectional (correlation)
   - Directional (regression)

Observed Variables

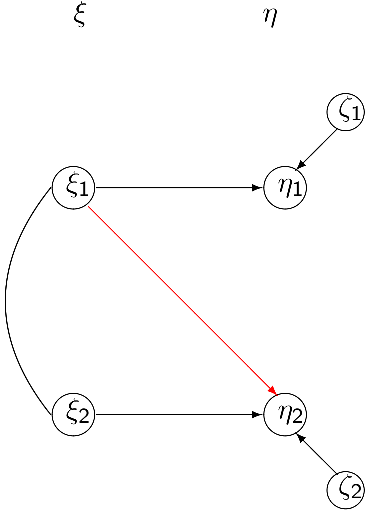


Latent Variables

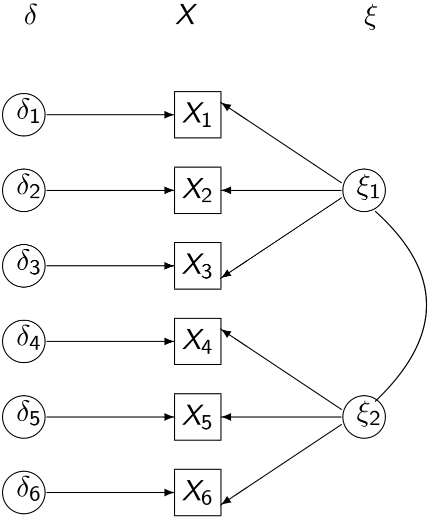
ξ η



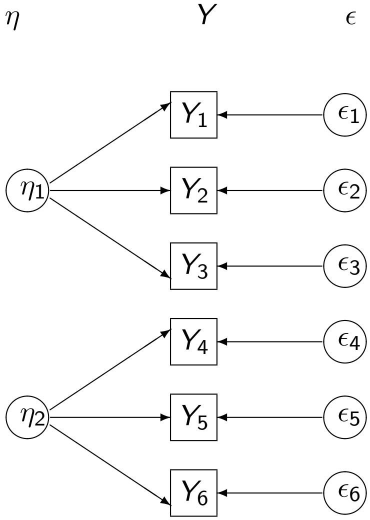
Theory: A regression model of latent variables



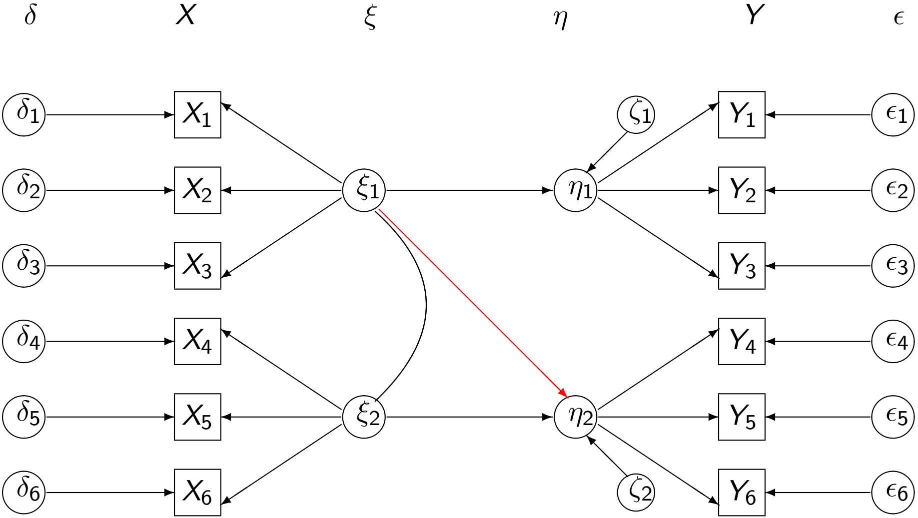
A measurement model for X



A measurement model for Y



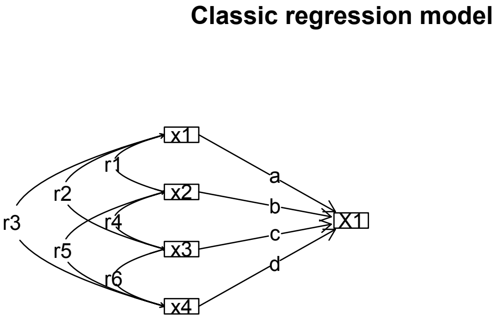
A complete structural model



Latent Variable Modeling

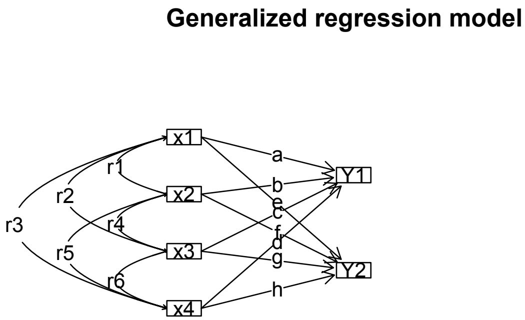
1. Requires measuring observed variables
    - Requires defining what is relevant and irrelevant to our theory.
    - Issues in quality of scale information, levels of measurement.
2. Formulating a measurement model of the data: estimating latent constructs
    - Perhaps based upon exploratory and then confirmatory factor analysis, definitely based upon theory.
    - Includes understanding the reliability of the measures.
3. Modeling the structure of the constructs
    - This is a combination of theory and fitting. Do the data fit the theory.
    - Comparison of models. Does one model fit better than alternative models?

The classic regression model



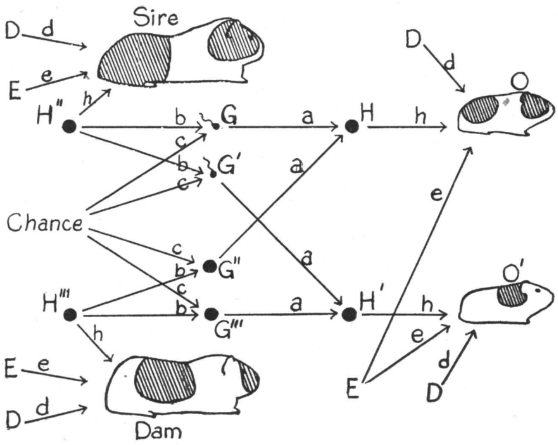
$$\hat{Y} = \beta_x x + \epsilon$$

The generalized regression model

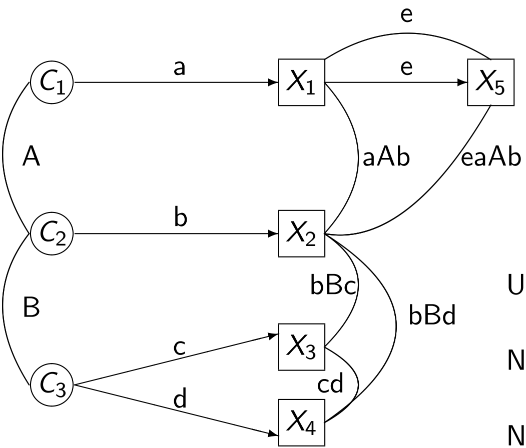


$$\hat{Y} = \beta_x x + \epsilon$$

Wright's Path model of inheritance in the Guinea Pig (Wright, 1921)



The basic rules of path analysis–think genetics



Up ... and over and down ...

No down and up

No double overs

Up ... and down ...

Parents cause children

children do not cause parents

Observed-observed

1. The Kerckhoff (1974) data set is used in the LISREL manual as an example of regression path models
   - "A sample of boys originally studied as ninth graders in 1969 was recontacted in 1974 to obtain information about high school performance and educational attainment."
   - 767 twelfth grade males
   - A study of background, aspiration and educational attainment
2. Variables
   - Intelligence
   - Number of siblings
   - Fathers Education
   - Fathers's occupation
   - Grades
   - Educational expectation
   - Occupational aspiration
3. Correlation matrix is available in the sem (Fox, Nie & Byrnes, 2013) package.

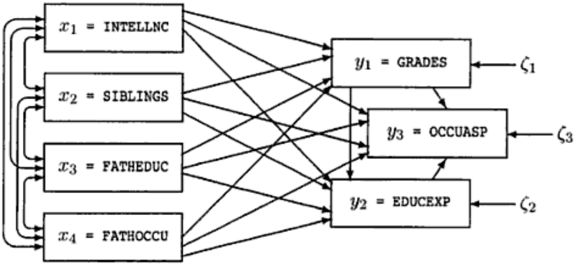
The Kerckhoff correlation matrix (N=767)

```
> R.kerch
             Intelligence Siblings FatherEd FatherOcc Grades EducExp OccupAsp
Intelligence        1.000   -0.100    0.277     0.250  0.572   0.489    0.335
Siblings           -0.100    1.000   -0.152    -0.108 -0.105  -0.213   -0.153
FatherEd            0.277   -0.152    1.000     0.611  0.294   0.446    0.303
FatherOcc           0.250   -0.108    0.611     1.000  0.248   0.410    0.331
Grades              0.572   -0.105    0.294     0.248  1.000   0.597    0.478
EducExp             0.489   -0.213    0.446     0.410  0.597   1.000    0.651
OccupAsp            0.335   -0.153    0.303     0.331  0.478   0.651    1.000
>
```

Conceptual Kerckhoff model

1. Background variables
   - Intelligence
   - Number of siblings
   - Fathers Education
   - Fathers's occupation
2. Intermediate variables
   - Grades
   - Educational expectation
3. Final outcomes
   - Occupational aspiration

The model (From the LISREL manual)



Matrix regression

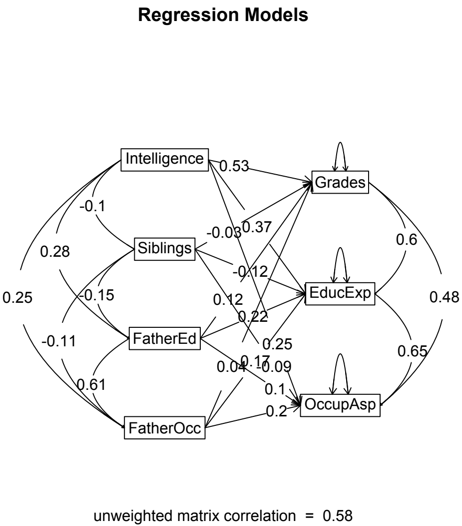
1. Most regression examples (and functions) use raw data
   - $\hat{Y} = X\beta + \epsilon$
   - $beta = (X'X)^{-1}X'Y$
   - lm(y ~x)
2. Regression is just solving the matrix equation
   - $\beta = R^{-1}r_{xy}$
   - mat.regress(R,x,y) (deprecated)
   - setCor(y,x,R) (recommended)

Simple regression 4 predictors, 3 criteria

setCor(y=5:7,x=1:4,data=R.kerch)

```
Call: setCor(y = 5:7, x = 1:4, data = R.kerch)

Multiple Regression from matrix input

Beta weights
             Grades EducExp OccupAsp
Intelligence   0.53    0.37     0.25
Siblings      -0.03   -0.12    -0.09
FatherEd       0.12    0.22     0.10
FatherOcc      0.04    0.17     0.20

Multiple R
  Grades  EducExp OccupAsp
    0.59     0.61     0.44
multiple R2
  Grades  EducExp OccupAsp
    0.35     0.38     0.19
```

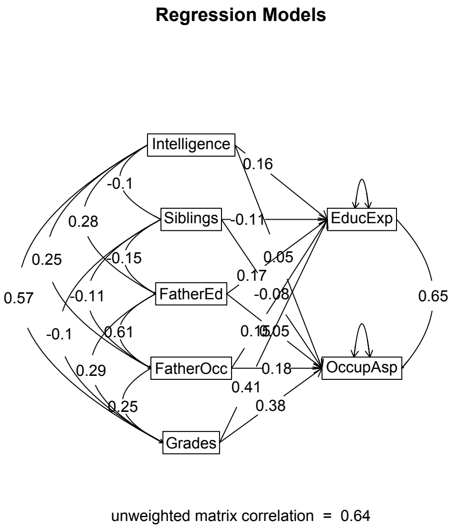
The paths (From setCor)



More complicated regression

```
> setCor(y=6:7,x=1:5,data=R.kerch)


Call: setCor(y = 6:7, x = 1:5, data = R.kerch)

Multiple Regression from matrix input

Beta weights
             EducExp OccupAsp
Intelligence    0.16     0.05
Siblings       -0.11    -0.08
FatherEd        0.17     0.05
FatherOcc       0.15     0.18
Grades          0.41     0.38

Multiple R
 EducExp OccupAsp
    0.70     0.54
multiple R2
 EducExp OccupAsp
    0.48     0.29
```

The paths (From `setCor`)



Predicting occupational aspirations from the intermediate set

```
Call: setCor(y = 7, x = 5:6, data = R.kerch)
Multiple Regression from matrix input

Beta weights
        OccupAsp
Grades      0.14
EducExp     0.57

Multiple R
     [,1]
[1,] 0.66
multiple R2
     [,1]
[1,] 0.44

 Unweighted multiple R
OccupAsp
    0.63
 Unweighted multiple R2
OccupAsp
     0.4
```

Predicting occupational aspirations from the entire set

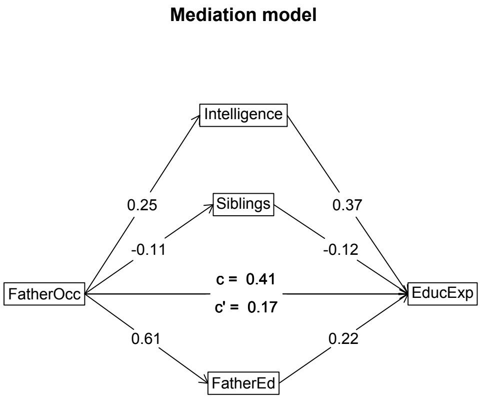
```
> setCor(y=7,x=1:6,data=R.kerch)

Call: setCor(y = 7, x = 1:6, data = R.kerch)

Multiple Regression from matrix input

Beta weights
             OccupAsp
Intelligence    -0.04
Siblings        -0.02
FatherEd        -0.04
FatherOcc        0.10
Grades           0.16
EducExp          0.55

Multiple R
OccupAsp
    0.67

Multiple R2
OccupAsp
    0.44
```

We can also examine mediation using regression (and the `mediate` function).

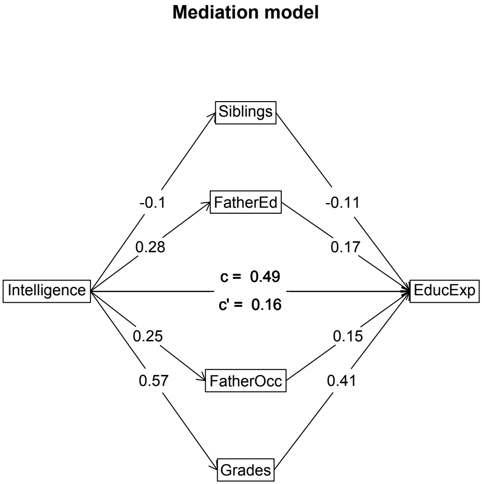
mediate(6,x=4,m=1:3,data=R.kerch)

```
> mediate(6,x=4,m=1:3,data=R.kerch,n.obs=767)
The data matrix was simulated based upon the specified number of subjects and t

Mediation analysis
Call: mediate(y = 6, x = 4, m = 1:3, data = R.kerch, n.obs = 767)

The DV (Y) was  EducExp . The IV (X) was  FatherOcc . The mediating variable(s)
Total Direct effect(c) of  FatherOcc  on  EducExp  =  0.41   S.E. =  0.03  t di
Direct effect of  FatherOcc  on  EducExp  removing  Intelligence Siblings Fathe
                                                S.E. =  0.04  t direct =  4.63
Indirect effect (c') of  FatherOcc  on  EducExp  through  Intelligence Siblings

Mean bootstrapped indirect effect =  0.28  with standard error =  0.03
                                        Lower CI =  0.23    Upper CI =  0.34
Summary of a, b, and ab estimates and ab confidence intervals
                 a     b   ab mean.ab ci.ablower ci.abupper
Intelligence  0.25  0.37 0.09    0.10       0.07       0.13
Siblings     -0.11 -0.12 0.01    0.02       0.01       0.03
FatherEd      0.61  0.22 0.13    0.17       0.12       0.22

ratio of indirect to total effect=   0.59
```

The mediated paths (From `mediate`)



An alternative mediated model (From `mediate`)



Can use sem functions (in either sem or lavaan) to estimate the mediation model

1. Treat all variables as observed (fixed)
   - Specify a limited number of paths rather than the full regression model
2. *sem* commands
   - either in RAM (path) notation or
   - causal notation
3. *lavaan* commands similar to causal notation of sem and the commercial programs Mplus and EQS
   - Output can mimic either MPlus or LISREL
   - Commands can be translated directly from lavaan to MPlus

Kerckhoff-Kenny path analysis (modified from sem help page) to just predict the DVs)

```
model.kerch <- specifyModel()
    Intelligence -> Grades,       gam51
    Siblings -> Grades,           gam52
    FatherEd -> Grades,           gam53
    FatherOcc -> Grades,          gam54
    Intelligence -> EducExp,      gam61
    Siblings -> EducExp,          gam62
    FatherEd -> EducExp,          gam63
    FatherOcc -> EducExp,         gam64
     Intelligence -> OccupAsp,     gam71
    Siblings -> OccupAsp,         gam72
    FatherEd -> OccupAsp,         gam73
    FatherOcc -> OccupAsp,        gam74
  # Grades -> EducExp,            beta65
   # Grades -> OccupAsp,           beta75
 #   EducExp -> OccupAsp,          beta76

sem.kerch <- sem(model.kerch, R.kerch, 737, fixed.x=c('Intelligence','Siblings',
    'FatherEd','FatherOcc'))
summary(sem.kerch)
```

Fixed path model – not modeling the DV correlations

```
Model Chisquare =  411.72   Df =  3 Pr(>Chisq) = 6.4133e-89
Chisquare (null model) =  1664.3   Df =  21
Goodness-of-fit index =  0.85747
Adjusted goodness-of-fit index =  -0.33031
RMSEA index =  0.43024   90% CI: (0.39572, 0.46581)
Bentler-Bonnett NFI =  0.75262
Tucker-Lewis NNFI =  -0.74103
Bentler CFI =  0.75128
SRMR =  0.099759
AIC =  441.72
AICc =  412.38
BIC =  510.76
CAIC =  388.91

Normalized Residuals
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.000   0.000   0.000   0.956   0.000  10.100

R-square for Endogenous Variables
 Grades  EducExp OccupAsp
 0.3490   0.3765   0.1930
```

With path coefficients of

```
Parameter Estimates
            Estimate  Std Error z value  Pr(>|z|)
gam51        0.525902 0.031182  16.86530 8.0987e-64 Grades <--- Intelligence
gam52       -0.029942 0.030149  -0.99314 3.2064e-01 Grades <--- Siblings
gam53        0.118966 0.038259   3.10951 1.8740e-03 Grades <--- FatherEd
gam54        0.040603 0.037785   1.07456 2.8257e-01 Grades <--- FatherOcc
gam61        0.373339 0.030517  12.23376 2.0521e-34 EducExp <--- Intelligence
gam62       -0.123910 0.029506  -4.19954 2.6745e-05 EducExp <--- Siblings
gam63        0.220918 0.037442   5.90022 3.6302e-09 EducExp <--- FatherEd
gam64        0.168302 0.036979   4.55125 5.3328e-06 EducExp <--- FatherOcc
gam71        0.248827 0.034718   7.16718 7.6561e-13 OccupAsp <--- Intelligence
gam72       -0.091653 0.033567  -2.73047 6.3245e-03 OccupAsp <--- Siblings
gam73        0.098869 0.042596   2.32109 2.0282e-02 OccupAsp <--- FatherEd
gam74        0.198486 0.042069   4.71809 2.3807e-06 OccupAsp <--- FatherOcc
V[Grades]    0.650995 0.033935  19.18333 5.1010e-82 Grades <--> Grades
V[EducExp]   0.623511 0.032503  19.18333 5.1010e-82 EducExp <--> EducExp
V[OccupAsp]  0.806964 0.042066  19.18333 5.1010e-82 OccupAsp <--> OccupAsp
```

Note, we are not modeling the DV correlations so the residuals will be large

Residuals from this model

```
> lowerMat(resid(sem.kerch))
             Intll Sblng FthrE FthrO Grads EdcEx OccpA
Intelligence 0.00
Siblings     0.00  0.00
FatherEd     0.00  0.00  0.00
FatherOcc    0.00  0.00  0.00  0.00
Grades       0.00  0.00  0.00  0.00  0.00
EducExp      0.00  0.00  0.00  0.00  0.26  0.00
OccupAsp     0.00  0.00  0.00  0.00  0.25  0.38  0.00
```

fixed sem = regression

Compare the coefficients from this sem with the regression β values

```
round(sem.kerch$coeff,3)
      gam51       gam52       gam53       gam54       gam61       gam62       gam63
       gam64       gam71       gam72       gam73     gam74
      0.526      -0.030       0.119       0.041       0.373      -0.124       0.221
      0.168       0.249      -0.092       0.099       0.198
     V[Grades]  V[EducExp]  V[OccupAsp]
    0.651       0.624       0.807

  mr.kk <- mat.regress(data=R.kerch,x=c(1:4),y=c(5:7))

  > round(as.vector(mr.kk$beta),3)
 [1]  0.526 -0.030  0.119  0.041  0.373 -0.124  0.221
  0.168  0.249 -0.092  0.099  0.198
```

More complicated regression

1. Able to model the intercorrelations of the Y variables
   - This treats the Ys as both predictors and predicted
2. Able to let some of the Ys be part of the regression model

Complete Kerckhoff-Kenny path analysis (taken from sem help page)

```
model.kerch1 <- specifyModel()
    Intelligence -> Grades,        gam51
    Siblings -> Grades,            gam52
    FatherEd -> Grades,            gam53
    FatherOcc -> Grades,           gam54
    Intelligence -> EducExp,       gam61
    Siblings -> EducExp,           gam62
    FatherEd -> EducExp,           gam63
    FatherOcc -> EducExp,          gam64
    Grades -> EducExp,             beta65
    Intelligence -> OccupAsp,      gam71
    Siblings -> OccupAsp,          gam72
    FatherEd -> OccupAsp,          gam73
    FatherOcc -> OccupAsp,         gam74
    Grades -> OccupAsp,            beta75
    EducExp -> OccupAsp,           beta76

sem.kerch1 <- sem(model.kerch1, R.kerch, 737, fixed.x=c('Intelligence','Siblings',
    'FatherEd','FatherOcc'))
summary(sem.kerch1)
```

sem output

```
 Model Chisquare =  3.2685e-13   Df =  0 Pr(>Chisq) = NA
 Chisquare (null model) =  1664.3   Df =  21
 Goodness-of-fit index =  1
 AIC =  36
 AICc =  0.95265
 BIC =  118.85
 CAIC =  3.2685e-13
 Normalized Residuals
    Min.  1st Qu.    Median      Mean  3rd Qu.      Max.
-4.26e-15 -1.35e-15  0.00e+00 -4.17e-16  0.00e+00  1.49e-15
 Parameter Estimates
           Estimate  Std Error z value  Pr(>|z|)
gam51        0.525902 0.031182  16.86530 8.0987e-64 Grades <--- Intelligence
gam52       -0.029942 0.030149  -0.99314 3.2064e-01 Grades <--- Siblings
gam53        0.118966 0.038259   3.10951 1.8740e-03 Grades <--- FatherEd
gam54        0.040603 0.037785   1.07456 2.8257e-01 Grades <--- FatherOcc
gam61        0.160270 0.032710   4.89979 9.5940e-07 EducExp <--- Intelligence
gam62       -0.111779 0.026876  -4.15899 3.1966e-05 EducExp <--- Siblings
gam63        0.172719 0.034306   5.03461 4.7882e-07 EducExp <--- FatherEd
gam64        0.151852 0.033688   4.50758 6.5571e-06 EducExp <--- FatherOcc
beta65       0.405150 0.032838  12.33799 5.6552e-35 EducExp <--- Grades
gam71       -0.039405 0.034500  -1.14215 2.5339e-01 OccupAsp <--- Intelligence
gam72       -0.018825 0.028222  -0.66700 5.0477e-01 OccupAsp <--- Siblings
gam73       -0.041333 0.036216  -1.14126 2.5376e-01 OccupAsp <--- FatherEd
gam74        0.099577 0.035446   2.80924 4.9658e-03 OccupAsp <--- FatherOcc
beta75       0.157912 0.037443   4.21738 2.4716e-05 OccupAsp <--- Grades
beta76       0.549593 0.038260  14.36486 8.5976e-47 OccupAsp <--- EducExp
V[Grades]    0.650995 0.033935  19.18333 5.1010e-82 Grades <--> Grades
V[EducExp]   0.516652 0.026932  19.18333 5.1010e-82 EducExp <--> EducExp
V[OccupAsp]  0.556617 0.029016  19.18333 5.1010e-82 OccupAsp <--> OccupAsp
```

Note that these models give different path coefficients

```
> round(sem.kerch$coeff,3)
     gam51       gam52       gam53       gam54       gam61       gam62       gam63       gam64
      gam71       gam72       gam73
     0.526      -0.030       0.119       0.041       0.373      -0.124       0.221       0.168
     0.249      -0.092       0.099
     gam74   V[Grades]  V[EducExp] V[OccupAsp]
     0.198       0.651       0.624       0.807
> round(sem.kerch1$coeff,3)
     gam51       gam52       gam53       gam54       gam61       gam62       gam63       gam64
     beta65       gam71       gam72
     0.526      -0.030       0.119       0.041       0.160      -0.112       0.173       0.152
     0.405      -0.039      -0.019
     gam73       gam74      beta75      beta76   V[Grades]  V[EducExp] V[OccupAsp]
    -0.041       0.100       0.158       0.550       0.651       0.517       0.557

> round(mr.kk$beta,2)
             Grades EducExp OccupAsp
Intelligence   0.53    0.37     0.25
Siblings      -0.03   -0.12    -0.09
FatherEd       0.12    0.22     0.10
FatherOcc      0.04    0.17     0.20
```

A latent variable structural model

1. Taken from the LISREL User's reference guide
2. Data from Caslyn and Kenny (1977)
   - Self-concept of ability and perceived evaluation of others: Cause or effect of academic achievement
3. Variables
   - self concept
   - parental evaluation
   - teacher evaluation
   - friend evaluation
   - educational aspiration
   - college plans

Caslyn and Kenny data

	self	parent	teacher	friend	edu_asp	college
self_concept	1.00	0.73	0.70	0.58	0.46	0.56
parental_eval	0.73	1.00	0.68	0.61	0.43	0.52
teacher_eval	0.70	0.68	1.00	0.57	0.40	0.48
friend_eval	0.58	0.61	0.57	1.00	0.37	0.41
edu_aspir	0.46	0.43	0.40	0.37	1.00	0.72
college_plans	0.56	0.52	0.48	0.41	0.72	1.00

Creating the model using `structure.diagam`

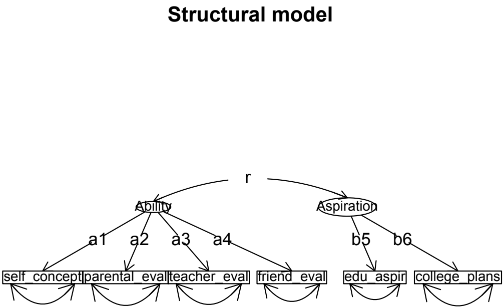
```
fx <- structure.list(6,list(c(1:4),c(5:6)),item.labels = rownames(ability),
                     f.labels=c("Ability","Aspiration"))
mod.edu <- structure.diagram(fx,"r",title="Lisrel example 3.2",
                        errors=TRUE,lr=FALSE,cex=.8)
fx
```

```
fx
              Ability Aspiration
self_concept  "a1"    "0"
parental_eval "a2"    "0"
teacher_eval  "a3"    "0"
friend_eval   "a4"    "0"
edu_aspir     "0"     "b5"
college_plans "0"     "b6"
```

The sem commands are in the mod.edu object

```
mod.edu
      Path                                 Parameter Value
 [1,] "Ability->self_concept"              "a1"      NA
 [2,] "Ability->parental_eval"             "a2"      NA
 [3,] "Ability->teacher_eval"              "a3"      NA
 [4,] "Ability->friend_eval"               "a4"      NA
 [5,] "Aspiration->edu_aspir"              "b5"      NA
 [6,] "Aspiration->college_plans"          "b6"      NA
 [7,] "self_concept<->self_concept"        "x1e"     NA
 [8,] "parental_eval<->parental_eval"      "x2e"     NA
 [9,] "teacher_eval<->teacher_eval"        "x3e"     NA
[10,] "friend_eval<->friend_eval"          "x4e"     NA
[11,] "edu_aspir<->edu_aspir"              "x5e"     NA
[12,] "college_plans<->college_plans"      "x6e"     NA
[13,] "Aspiration<->Ability"               "rF2F1"   NA
[14,] "Ability<->Ability"                  NA        "1"
[15,] "Aspiration<->Aspiration"            NA        "1"
```

A model of the Caslyn-Kenny (1997) data set



```
ability <- as.matrix(ability) #sem requires matrix input
sem.edu <- sem(mod=mod.edu,S=ability,N=556)
summary(sem.edu)
```

```
Model Chisquare =  9.2557   Df =  8 Pr(>Chisq) = 0.32118
 Chisquare (null model) =  1832   Df =  15
 Goodness-of-fit index =  0.99443
 Adjusted goodness-of-fit index =  0.98537
 RMSEA index =  0.016817   90% CI: (NA, 0.054321)
 Bentler-Bonnett NFI =  0.99495
 Tucker-Lewis NNFI =  0.9987
 Bentler CFI =  0.99931
 SRMR =  0.012011
 AIC =  35.256
 AICc =  9.9273
 BIC =  91.426
 CAIC =  -49.31

 Normalized Residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.4410 -0.1870  0.0000 -0.0131  0.2110  0.5330

 R-square for Endogenous Variables
 self_concept parental_eval  teacher_eval   friend_eval     edu_aspir college_plans
       0.7451        0.7213        0.6482        0.4834        0.6008        0.8629
```

With parameter estimates

```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
a1    0.86320  0.035145  24.5612 3.2846e-133 self_concept <--- Ability
a2    0.84932  0.035450  23.9582 7.5937e-127 parental_eval <--- Ability
a3    0.80509  0.036405  22.1149 2.2725e-108 teacher_eval <--- Ability
a4    0.69527  0.038634  17.9964  2.0795e-72 friend_eval <--- Ability
b5    0.77508  0.040357  19.2058  3.3077e-82 edu_aspir <--- Aspiration
b6    0.92893  0.039410  23.5712 7.6153e-123 college_plans <--- Aspiration
x1e   0.25488  0.023367  10.9075  1.0617e-27 self_concept <--> self_concept
x2e   0.27865  0.024128  11.5491  7.4600e-31 parental_eval <--> parental_eval
x3e   0.35184  0.026919  13.0703  4.8660e-39 teacher_eval <--> teacher_eval
x4e   0.51660  0.034725  14.8768  4.6594e-50 friend_eval <--> friend_eval
x5e   0.39924  0.038196  10.4525  1.4266e-25 edu_aspir <--> edu_aspir
x6e   0.13709  0.043505   3.1511  1.6264e-03 college_plans <--> college_plans
rF2F1 0.66637  0.030954  21.5276 8.5783e-103 Ability <--> Aspiration
```

Three competing models

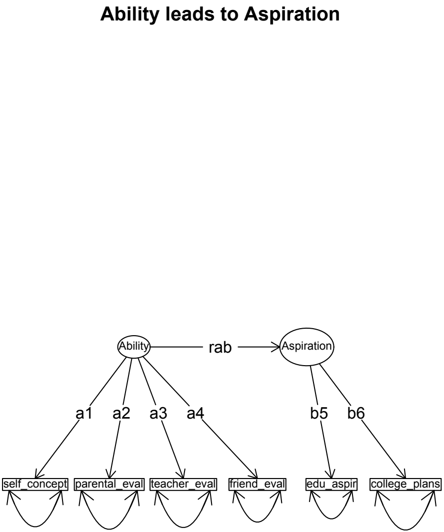
1. Ability and aspirations are correlated
   - r = .66
2. Ability causes aspirations
   - beta = .89
3. Aspirations cause ability
   - beta = .89

Create the model where ability lead to aspirations

```
phi <- phi.list(2,c(2))
phi
mod.edu <- structure.diagram(fx,phi,title="Aspiration leads to ability",
                        errors=TRUE,lr=FALSE,cex=.7)
```

```
mod.edu
      Path                                 Parameter Value
 [1,] "Ability->self_concept"              "a1"      NA
 [2,] "Ability->parental_eval"             "a2"      NA
 [3,] "Ability->teacher_eval"              "a3"      NA
 [4,] "Ability->friend_eval"               "a4"      NA
 [5,] "Aspiration->edu_aspir"              "b5"      NA
 [6,] "Aspiration->college_plans"          "b6"      NA
 [7,] "self_concept<->self_concept"        "x1e"     NA
 [8,] "parental_eval<->parental_eval"      "x2e"     NA
 [9,] "teacher_eval<->teacher_eval"        "x3e"     NA
[10,] "friend_eval<->friend_eval"          "x4e"     NA
[11,] "edu_aspir<->edu_aspir"              "x5e"     NA
[12,] "college_plans<->college_plans"      "x6e"     NA
[13,] "Ability ->Aspiration"               "rF1F2"   NA
[14,] "Ability<->Ability"                  NA        "1"
[15,] "Aspiration<->Aspiration"            NA        "1"
```

Ability causes aspiration



Fit statistics are identical

```
> sem.edu <- sem(mod=mod.edu,S=ability,N=556)
> summary(sem.edu)

summary(sem.edu)

 Model Chisquare =  9.2557   Df =  8 Pr(>Chisq) = 0.32118
 Chisquare (null model) =  1832   Df =  15
 Goodness-of-fit index =  0.99443
 Adjusted goodness-of-fit index =  0.98537
 RMSEA index =  0.016817   90% CI: (NA, 0.054321)
 Bentler-Bonnett NFI =  0.99495
 Tucker-Lewis NNFI =  0.9987
 Bentler CFI =  0.99931
 SRMR =  0.012011
 AIC =  35.256
 AICc =  9.9273
 BIC =  91.426
 CAIC =  -49.31

 Normalized Residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.4410 -0.1870  0.0000 -0.0131  0.2110  0.5330

 R-square for Endogenous Variables
 self_concept parental_eval  teacher_eval   friend_eval    Aspiration     edu_aspir college_plans
       0.7451        0.7213        0.6482        0.4834        0.4440        0.6008        0.8629
```

But the paths are different

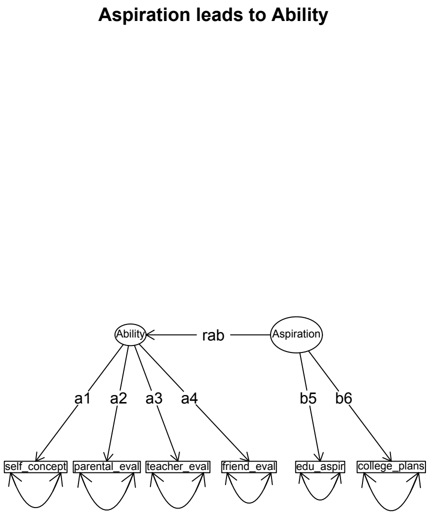
```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
a1    0.86320  0.035145  24.5612 3.2848e-133 self_concept <--- Ability
a2    0.84932  0.035450  23.9582 7.5919e-127 parental_eval <--- Ability
a3    0.80509  0.036405  22.1149 2.2732e-108 teacher_eval <--- Ability
a4    0.69527  0.038634  17.9964  2.0794e-72 friend_eval <--- Ability
b5    0.57792  0.030630  18.8678  2.0977e-79 edu_aspir <--- Aspiration
b6    0.69263  0.037979  18.2370  2.6257e-74 college_plans <--- Aspiration
x1e   0.25488  0.023367  10.9075  1.0617e-27 self_concept <--> self_concept
x2e   0.27865  0.024128  11.5491  7.4610e-31 parental_eval <--> parental_eval
x3e   0.35184  0.026919  13.0703  4.8654e-39 teacher_eval <--> teacher_eval
x4e   0.51660  0.034725  14.8768  4.6595e-50 friend_eval <--> friend_eval
x5e   0.39924  0.038196  10.4525  1.4266e-25 edu_aspir <--> edu_aspir
x6e   0.13709  0.043505   3.1511  1.6264e-03 college_plans <--> college_plans
rF1F2 0.89371  0.074673  11.9683  5.2068e-33 Aspiration <--- Ability

 Iterations =  30
```

Let aspirations cause ability

```
> phi[1,2] <- phi[2,1]
> phi[2,1] <- "0"
> mod.edu <- structure.diagram(fx,phi,main="Aspiration leads to Ability",errors=
>mod.edu
```

```
> mod.edu
      Path                                Parameter Value
 [1,] "Ability->self_concept"             "a1"      NA
 [2,] "Ability->parental_eval"            "a2"      NA
 [3,] "Ability->teacher_eval"             "a3"      NA
 [4,] "Ability->friend_eval"              "a4"      NA
 [5,] "Aspiration->edu_aspir"             "b5"      NA
 [6,] "Aspiration->college_plans"         "b6"      NA
 [7,] "self_concept<->self_concept"       "x1e"     NA
 [8,] "parental_eval<->parental_eval"     "x2e"     NA
 [9,] "teacher_eval<->teacher_eval"       "x3e"     NA
[10,] "friend_eval<->friend_eval"         "x4e"     NA
[11,] "edu_aspir<->edu_aspir"             "x5e"     NA
[12,] "college_plans<->college_plans"     "x6e"     NA
[13,] "Aspiration<-Ability"               "rF2F1"   NA
[14,] "Ability<->Ability"                 NA        "1"
[15,] "Aspiration<->Aspiration"           NA        "1"
```

Aspiration leads to ability



Fits are still identical

```
> sem.edu <- sem(mod=mod.edu,S=ability,N=556)
> summary(sem.edu)
 Model Chisquare =  9.2557   Df =  8 Pr(>Chisq) = 0.32118
 Chisquare (null model) =  1832   Df =  15
 Goodness-of-fit index =  0.99443
 Adjusted goodness-of-fit index =  0.98537
 RMSEA index =  0.016817   90% CI: (NA, 0.054321)
 Bentler-Bonnett NFI =  0.99495
 Tucker-Lewis NNFI =  0.9987
 Bentler CFI =  0.99931
 SRMR =  0.012011
 AIC =  35.256
 AICc =  9.9273
 BIC =  91.426
 CAIC =  -49.31

 Normalized Residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-0.4410 -0.1870  0.0000 -0.0131  0.2110  0.5330

 R-square for Endogenous Variables
 self_concept parental_eval  teacher_eval   friend_eval    Aspiration     edu_a
       0.7451        0.7213        0.6482        0.4834        0.4440         0.
```

And the crucial coefficient is different

```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
a1    0.86320  0.035145  24.5612 3.2848e-133 self_concept <--- Ability
a2    0.84932  0.035450  23.9582 7.5919e-127 parental_eval <--- Ability
a3    0.80509  0.036405  22.1149 2.2732e-108 teacher_eval <--- Ability
a4    0.69527  0.038634  17.9964  2.0794e-72 friend_eval <--- Ability
b5    0.57792  0.030630  18.8678  2.0977e-79 edu_aspir <--- Aspiration
b6    0.69263  0.037979  18.2370  2.6257e-74 college_plans <--- Aspiration
x1e   0.25488  0.023367  10.9075  1.0617e-27 self_concept <--> self_concept
x2e   0.27865  0.024128  11.5491  7.4610e-31 parental_eval <--> parental_eval
x3e   0.35184  0.026919  13.0703  4.8654e-39 teacher_eval <--> teacher_eval
x4e   0.51660  0.034725  14.8768  4.6595e-50 friend_eval <--> friend_eval
x5e   0.39924  0.038196  10.4525  1.4266e-25 edu_aspir <--> edu_aspir
x6e   0.13709  0.043505   3.1511  1.6264e-03 college_plans <--> college_plans
rF2F1 0.89371  0.074673  11.9683  5.2068e-33 Aspiration <--- Ability
```

Compare the three models

	correlated	asp	abil
a1	0.86	0.64	0.86
a2	0.85	0.63	0.85
a3	0.81	0.60	0.81
a4	0.70	0.52	0.70
b5	0.78	0.78	0.58
b6	0.93	0.93	0.69
x1e	0.25	0.25	0.25
x2e	0.28	0.28	0.28
x3e	0.35	0.35	0.35
x4e	0.52	0.52	0.52
x5e	0.40	0.40	0.40
x6e	0.14	0.14	0.14
rF2F1	0.67	0.89	0.89

1. Although fits were identical
2. Paths differ as a function of presumed influence
3. Which solution is correct?
4. Is this even possible to answer?

lavaan syntax is perhaps easier (correlated latents)

R code

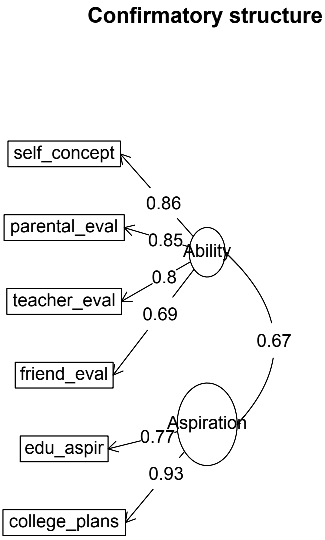
```
model <- "Ability =~ self_concept + parental_eval +
                              teacher_eval + friend_eval
         Aspiration =~ edu_aspir + college_plans"
fit <- sem(model,sample.cov=casslyn,fixed.x=TRUE,sample.nobs=556,
       std.lv=TRUE,mimic="eqs")   #note that the default is lavaan
```

```
lavaan (0.5-17) converged normally after  22 iterations

  Number of observations                           556

  Estimator                                         ML
  Minimum Function Test Statistic                9.256
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.321

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

lavaan output (continued)

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  Ability =~
    self_concept      0.863    0.035   24.561    0.000
    parental_eval     0.849    0.035   23.958    0.000
    teacher_eval      0.805    0.036   22.115    0.000
    friend_eval       0.695    0.039   17.996    0.000
  Aspiration =~
    edu_aspir         0.775    0.040   19.206    0.000
    college_plans     0.929    0.039   23.571    0.000

Covariances:
  Ability ~~
    Aspiration        0.666    0.031   21.528    0.000

Variances:
    self_concept      0.255    0.023   10.907    0.000
    parental_eval     0.279    0.024   11.549    0.000
    teacher_eval      0.352    0.027   13.070    0.000
    friend_eval       0.517    0.035   14.877    0.000
    edu_aspir         0.399    0.038   10.453    0.000
    college_plans     0.137    0.044    3.151    0.002
    Ability           1.000
    Aspiration        1.000
```

A model of the Caslyn-Kenny (1997) data set using lavaan



lavaan syntax for regressions

R code

```
model <- "Ability =~ self_concept + parental_eval +
                              teacher_eval + friend_eval
         Aspiration =~ edu_aspir + college_plans
         Aspiration ~ Ability"
fit <- sem(model,sample.cov=casslyn,fixed.x=TRUE,sample.nobs=556,
       std.lv=TRUE,mimic="eqs")   #note that the default is lavaan
```

```
lavaan (0.5-17) converged normally after  22 iterations

  Number of observations                           556

  Estimator                                         ML
  Minimum Function Test Statistic                9.256
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.321

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
```

lavaan regression output, continued

```
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  Ability =~
    self_concept      0.863    0.035   24.561    0.000
    parental_eval     0.849    0.035   23.958    0.000
    teacher_eval      0.805    0.036   22.115    0.000
    friend_eval       0.695    0.039   17.996    0.000
  Aspiration =~
    edu_aspir         0.578    0.031   18.868    0.000
    college_plans     0.693    0.038   18.237    0.000

Regressions:
  Aspiration ~
    Ability           0.894    0.075   11.968    0.000

Variances:
    self_concept      0.255    0.023   10.907    0.000
    parental_eval     0.279    0.024   11.549    0.000
    teacher_eval      0.352    0.027   13.070    0.000
    friend_eval       0.517    0.035   14.877    0.000
    edu_aspir         0.399    0.038   10.453    0.000
    college_plans     0.137    0.044    3.151    0.002
    Ability           1.000
```

Implications of arrows

1. Need to fit alternative models
   - Create alternative plausible models
   - Create alternative implausible models (they will fit also).
2. Need to consider alternative representations
   - Try reversing arrows
3. Are there external variables (e.g., time) that allow one to choose between models?
4. Confirmation that a model fits does not confirm theoretical adequacy of the model.

Two fundamentally different types of observed variables

1. Observed variables can be "reflective" of the latent variables. They are "effect indicators".
   - Variables are caused by the latent variables.
   - Covariation between the variables are explained by the latent variables
2. Observed variables can be "causal indicators" or formative indicators that can directly effect the latent variable
   - Variables cause the "latent" variable
   - Covariation of the the observed variables is not modeled

Formative indicators Bollen (2002)

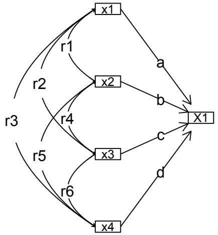
1. The correlational structure of formative indicators is independent of the loadings on a factor.
   - They are not locally independent
2. Examples of formative indicators Time spent in social interaction
   - Time spent with family, time spent with friends, time spent with coworkers.
   - These might in fact be negatively correlated even though total score is important.

Effect (reflective) indicators

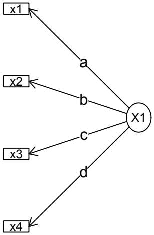
1. Test scores on various quantitative tests as effect indicators of trait
   - Feelings of self worth as effect indicators of self esteem
   - Ability items as indicators of ability
2. Correlational structure is a function of the path coefficients with latent variables
3. Variables are locally independent
   - (uncorrelated with each other when latent variable is partialled out)

Formative vs. Effect variables as regression versus factors

Formative Variables



Effect Variables



Confirmatory Factor Analysis as a way of testing factor models

1. Exploratory factor analysis (EFA) is a way of summarizing the relationships between variables
    - May have goodness of fit of factor model to the covariances but this is unusual
    - But all loadings are modeled
2. Confirmatory factor analysis (CFA) also models the data
    - But limits paths to specified paths, sets other to zero
    - Goodness of fit statistics are always produced
3. Blending CFA with regression is a full Structural Equation Model

Using sim.structural to create models

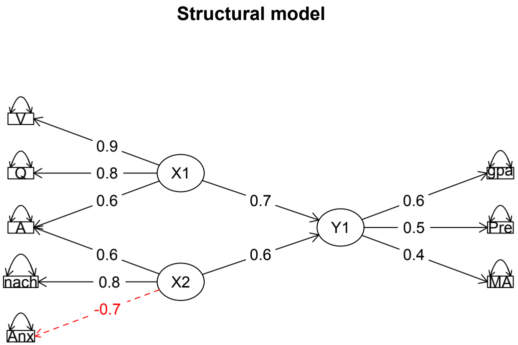
```
fx <-matrix(c( .9,.8,.6,rep(0,4),.6,.8,-.7),ncol=2)
fy <- matrix(c(.6,.5,.4),ncol=1)
rownames(fx) <- c("V","Q","A","nach","Anx")
rownames(fy)<- c("gpa","Pre","MA")
Phi <-matrix( c(1,0,.7,.0,1,.6,.7,.6,1),ncol=3)
gre.gpa <- sim.structural(fx,Phi,fy,n=1000)
gre.gpa
```

```
Call: sim.structural(fx = fx, Phi = Phi, fy = fy)

 $model (Population correlation matrix)
        V    Q     A  nach   Anx   gpa   Pre    MA
V    1.00 0.72  0.54  0.00  0.00  0.38  0.32  0.25
Q    0.72 1.00  0.48  0.00  0.00  0.34  0.28  0.22
A    0.54 0.48  1.00  0.48 -0.42  0.47  0.39  0.31
nach 0.00 0.00  0.48  1.00 -0.56  0.29  0.24  0.19
Anx  0.00 0.00 -0.42 -0.56  1.00 -0.25 -0.21 -0.17
gpa  0.38 0.34  0.47  0.29 -0.25  1.00  0.30  0.24
Pre  0.32 0.28  0.39  0.24 -0.21  0.30  1.00  0.20
MA   0.25 0.22  0.31  0.19 -0.17  0.24  0.20  1.00

$reliability (population reliability)
   V    Q    A nach  Anx  gpa  Pre   MA
0.81 0.64 0.72 0.64 0.49 0.36 0.25 0.16

$reliability (population reliability)
   V    Q    A nach  Anx  gpa  Pre   MA
0.81 0.64 0.72 0.64 0.49 0.36 0.25 0.16
```

```
> fx
> fy
> Phi

>  fx
     [,1] [,2]
V     0.9  0.0
Q     0.8  0.0
A     0.6  0.6
nach  0.0  0.8
Anx   0.0 -0.7
> Phi
     [,1] [,2] [,3]
[1,]  1.0  0.0  0.7
[2,]  0.0  1.0  0.6
[3,]  0.7  0.6  1.0
> fy
    [,1]
gpa  0.6
Pre  0.5
MA   0.4
```

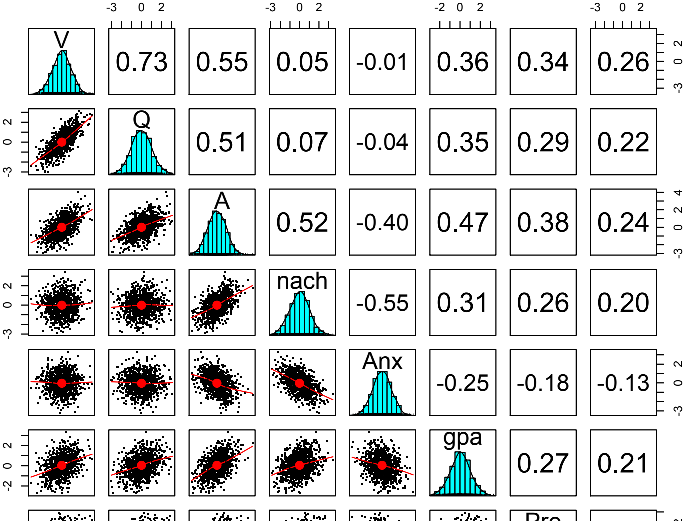
Create a figure (and write sem code for the sem package)

mod4 <- structure.diagram(fx,Phi,fy,errors=TRUE,e.size=.3)



Show the SPLOM

pairs.panels(gre.gpa$observed,pch=".")



Show the sem code – This uses RAM path notation for all arrows

```
> mod4

      Path            Parameter Value
 [1,] "X1->V"         "F1V"     NA
 [2,] "X1->Q"         "F1Q"     NA
 [3,] "X1->A"         "F1A"     NA
 [4,] "X2->A"         "F2A"     NA
 [5,] "X2->nach"      "F2nach"  NA
 [6,] "X2->Anx"       "F2Anx"   NA
 [7,] "V<->V"         "x1e"     NA
 [8,] "Q<->Q"         "x2e"     NA
 [9,] "A<->A"         "x3e"     NA
[10,] "nach<->nach"   "x4e"     NA
[11,] "Anx<->Anx"     "x5e"     NA
[12,] "Y1->gpa"       "Fygpa"   NA
[13,] "Y1->Pre"       "FyPre"   NA
[14,] "Y1->MA"        "FyMA"    NA
[15,] "gpa<->gpa"     "y1e"     NA
[16,] "Pre<->Pre"     "y2e"     NA
[17,] "MA<->MA"       "y3e"     NA
[18,] "X2<->X1"       "rF2F1"   NA
[19,] "X1->Y1"        "rX1Y1"   NA
[20,] "X2->Y1"        "rX2Y1"   NA
[21,] "X1<->X1"       NA        "1"
[22,] "X2<->X2"       NA        "1"
[23,] "Y1<->Y1"       NA        "1"
attr(,"class")
[1] "mod"
```

Can we recover the structure using a confirmatory factor model?

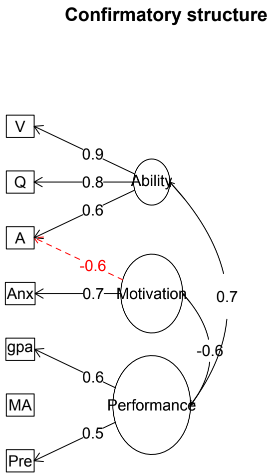
R code

```
model <- 'Ability =~ V + Q + A
               Motivation =~ A + Anx
               Performance =~ gpa + MA + Pre'
  fit <- sem(model,gre.gpa$observed,std.lv=TRUE)
  summary(fit)
  lavaan.diagram(fit)
```

lavaan output

```
lavaan (0.5-17) converged normally after  28 iterations

  Number of observations                          1000

  Estimator                                         ML
  Minimum Function Test Statistic               12.116
  Degrees of freedom                                10
  P-value (Chi-square)                           0.277

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  Ability =~
    V                 0.924    0.029   32.165    0.000
    Q                 0.830    0.029   28.263    0.000
    A                 0.614    0.033   18.540    0.000
  Motivation =~
    A                -0.593    0.038  -15.642    0.000
    Anx               0.660    0.044   14.990    0.000
  Performance =~
    gpa               0.570    0.036   16.001    0.000
    MA                0.341    0.035    9.637    0.000
    Pre               0.483    0.035   13.678    0.000

Covariances:
  Ability ~~
    Motivation       -0.033    0.051   -0.654    0.513
    Performance       0.748    0.039   19.386    0.000
```

Create a figure from the lavaan output)



But the data can also be modeled as full SEM model

R code

```
model <- 'Ability =~ V + Q + A
               Motivation =~ A + Anx
               Performance =~ gpa + MA + Pre
               Performance ~ Ability + Motivation'
  fit <- sem(model,gre.gpa$observed,std.lv=TRUE)
  summary(fit)
  standardizedSolution(fit)
  lavaan.diagram(fit)
```

lavaan solution for the regression model

```
standardizedSolution(fit)
           lhs op         rhs est.std    se       z pvalue
1      Ability =~           V   0.899 0.028  32.165  0.000
2      Ability =~           Q   0.810 0.029  28.263  0.000
3      Ability =~           A   0.593 0.032  18.540  0.000
4   Motivation =~           A  -0.573 0.037 -15.642  0.000
5   Motivation =~         Anx   0.672 0.045  14.990  0.000
6  Performance =~         gpa   0.579 0.198   2.932  0.003
7  Performance =~          MA   0.340 0.117   2.891  0.004
8  Performance =~         Pre   0.484 0.165   2.941  0.003
9  Performance  ~     Ability   0.729 0.252   2.889  0.004
10 Performance  ~  Motivation  -0.575 0.209  -2.749  0.006
11           V ~~           V   0.193 0.026   7.444  0.000
12           Q ~~           Q   0.344 0.025  13.704  0.000
13           A ~~           A   0.297 0.034   8.625  0.000
14         Anx ~~         Anx   0.548 0.053  10.301  0.000
15         gpa ~~         gpa   0.664 0.039  17.001  0.000
16          MA ~~          MA   0.885 0.041  21.435  0.000
17         Pre ~~         Pre   0.766 0.039  19.660  0.000
18     Ability ~~     Ability   1.000    NA      NA     NA
19  Motivation ~~  Motivation   1.000    NA      NA     NA
20 Performance ~~ Performance   0.110    NA      NA     NA
21     Ability ~~  Motivation  -0.033 0.051  -0.654  0.513
```

Measuring structure at two (or more) time points

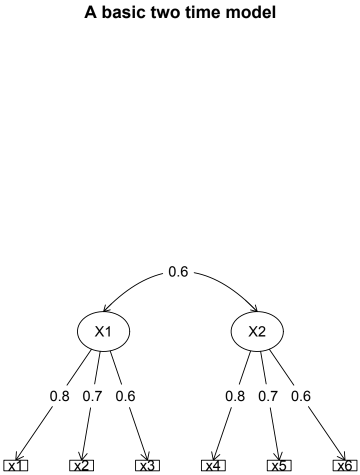
1. Is the structure the same
    - Structural Invariance (is the graph the same)
    - Measurement invariance (are the loadings the same)
    - Strong measurement invariance (are the item intercepts the same?)
    - Measuring change
2. Do the means change (is there growth)
    - This is the means of the latent trait, not the means of the items
3. Do the latent traits correlate across two or more occasions?
    - Just two occasions, can not separate trait from state effects
    - With > 2 occasions, can examine trait and state effects
4. Compare several different simulations

Create some basic data and add in some change

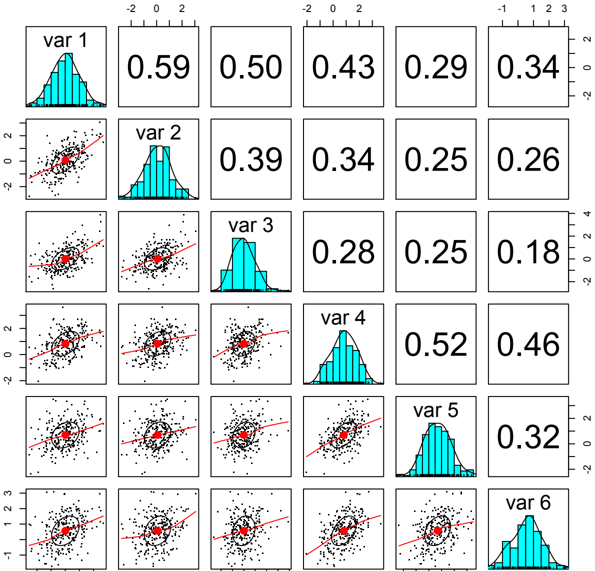
```
> set.seed(42)
 >  fx <- matrix(c(.8,.7,.6,rep(0,6),.8,.7,.6),ncol=2)
>  fx
     [,1] [,2]
[1,]  0.8  0.0
[2,]  0.7  0.0
[3,]  0.6  0.0
[4,]  0.0  0.8
[5,]  0.0  0.7
[6,]  0.0  0.6
> Phi <- matrix(c(1,.6,.6,1),ncol=2)
> Phi
     [,1] [,2]
[1,]  1.0  0.6
[2,]  0.6  1.0

> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
> x <- x.model$observed
> structure.diagram(fx,Phi,lr=FALSE,e.size=.3,main="A basic two time model")
> describe(x,skew=FALSE)

>  pairs.panels(x,pch=".")

   vars   n mean   sd median trimmed  mad   min  max range  skew kurtosis   se
1     1 250 0.01 0.97   0.06    0.01 0.92 -2.56 2.68  5.24  0.05     0.03 0.06
2     2 250 0.07 1.04   0.05    0.08 0.98 -2.76 3.07  5.83 -0.05    -0.04 0.07
3     3 250 0.00 0.98  -0.03   -0.04 0.97 -2.62 3.90  6.52  0.53     0.79 0.06
4     4 250 0.83 0.95   0.85    0.84 1.01 -2.04 3.64  5.68 -0.10    -0.35 0.06
5     5 250 0.70 1.02   0.69    0.68 1.00 -2.35 3.48  5.83  0.13     0.08 0.06
6     6 250 0.57 0.95   0.66    0.57 0.86 -1.71 3.07  4.79 -0.02    -0.22 0.06
```

1. Set the random seed
2. Create a one factor structure
3. Put in some change
4. Show the structural model
5. Describe it
6. Show the splom (with small pch)

A basic two occasion trait model



Splom of 6 basic variables



Multiple models

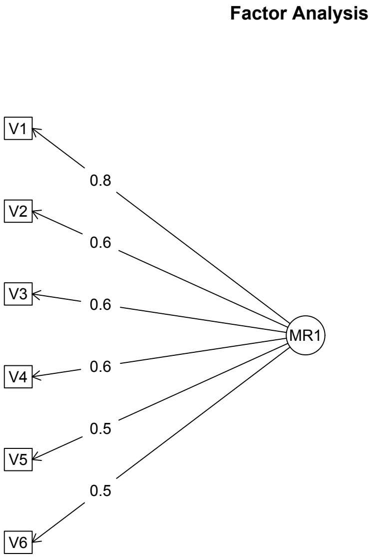
1. Ignore time, are the data congeneric?
    - Are they all measures of the same thing?
    - This is a one factor model
2. Include time, do we recover a correlation across time?
    - Try a two factor model
    - Plot the resulting structure

A one factor model

```
> fa(x)

Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix) based upon correlation matrix
   MR1   h2   u2 com
1 0.76 0.57 0.43   1
2 0.65 0.42 0.58   1
3 0.56 0.31 0.69   1
4 0.64 0.41 0.59   1
5 0.50 0.25 0.75   1
6 0.50 0.25 0.75   1

                MR1
SS loadings    2.21
Proportion Var 0.37

Mean item complexity =  1
Test of the hypothesis that 1 factor is sufficient.
```

```
The degrees of freedom for the null model are  15
    and the objective function was  1.57
             with Chi Square of  387.63
The degrees of freedom for the model are 9
              and the objective function was  0.28

The root mean square of the residuals (RMSR) is  0.09
The df corrected root mean square of
                               the residuals is  0.12

The harmonic number of observations is  250
           with the empirical chi square  62.96  with p
The total number of observations was  250
                     with MLE Chi Square =  67.96  with

Tucker Lewis Index of factoring reliability =  0.736
RMSEA index =  0.164  and the 90 % confidence intervals
BIC =  18.27
Fit based upon off diagonal values = 0.94
Measures of factor score adequacy
                                                 MR1
Correlation of scores with factors              0.89
Multiple R square of scores with factors        0.79
Minimum correlation of possible factor scores   0.58
```

Fit a one factor model to the data fa.diagram(fa(x),sort=FALSE)



Try a two factor solution

```
> fa(x,2)
```

```
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix
    MR1   MR2   h2   u2 com
1  0.88 -0.01 0.76 0.24 1.0
2  0.66  0.02 0.45 0.55 1.0
3  0.58  0.00 0.33 0.67 1.0
4 -0.01  0.87 0.75 0.25 1.0
5 -0.01  0.60 0.36 0.64 1.0
6  0.10  0.48 0.29 0.71 1.1

                       MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.58
MR2 0.58 1.00
Measures of factor score adequacy
                                                  MR1  MR2
Correlation of scores with factors               0.91 0.90
Multiple R square of scores with factors         0.83 0.81
Minimum correlation of possible factor scores    0.67 0.62
```

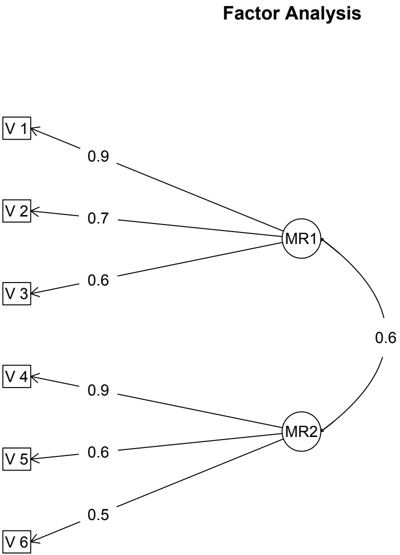
```
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  15  and
      the objective function was  1.57 with Chi Square
The degrees of freedom for the model are 4
               and the objective function was  0.01

The root mean square of the residuals (RMSR) is  0.02
The df corrected root mean square of the residuals is

The harmonic number of observations is  250
              with the empirical chi square  1.97  wit
The total number of observations was  250
              with MLE Chi Square =  2.51  with prob <

Tucker Lewis Index of factoring reliability =  1.015
RMSEA index =  0  and the 90 % confidence intervals ar
BIC =  -19.58
Fit based upon off diagonal values = 1
```

Compare the two solutions

```
Factor Analysis using method =  minres
Call: fa(r = x)
Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix) based upon correlation matrix
   MR1   h2   u2 com
1 0.76 0.57 0.43   1
2 0.65 0.42 0.58   1
3 0.56 0.31 0.69   1
4 0.64 0.41 0.59   1
5 0.50 0.25 0.75   1
6 0.50 0.25 0.75   1

                MR1
SS loadings    2.21
Proportion Var 0.37
```

```
Factor Analysis using method =  minres
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix) based upon corre
    MR1   MR2   h2   u2 com
1  0.88 -0.01 0.76 0.24 1.0
2  0.66  0.02 0.45 0.55 1.0
3  0.58  0.00 0.33 0.67 1.0
4 -0.01  0.87 0.75 0.25 1.0
5 -0.01  0.60 0.36 0.64 1.0
6  0.10  0.48 0.29 0.71 1.1

                       MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.58
MR2 0.58 1.00
```

EFA two factor solution



Now try three different sems (using lavaan)

1. Two correlated factors, free loadings
2. Two correlated factors, equal loadings across occasions
3. Two correlated factors, all loadings equal

A simple sem with a standardized solution using lavaan

R code

```
#factor model
mod2f <- 'F1 =~ V1 + V2 + V3
               F2 =~ V4 + V5 + V6
                  #correlation between factors
                 F1 ~~F2'# now fit it and summarize it
fit <- sem(mod2f, data=x,std.lv=TRUE)
summary(fit, fit.measures=TRUE)
standardizedSolution(fit)
```

```
   lhs op rhs est.std    se      z pvalue
1   F1 =~  V1   0.865 0.062 13.921      0
2   F1 =~  V2   0.681 0.063 10.756      0
3   F1 =~  V3   0.580 0.064  9.000      0
4   F2 =~  V4   0.838 0.067 12.599      0
5   F2 =~  V5   0.607 0.066  9.142      0
6   F2 =~  V6   0.558 0.067  8.348      0
7   F1 ~~  F2   0.602 0.061  9.887      0
8   V1 ~~  V1   0.251 0.068  3.713      0
9   V2 ~~  V2   0.536 0.064  8.443      0
10  V3 ~~  V3   0.664 0.068  9.763      0
11  V4 ~~  V4   0.297 0.077  3.885      0
12  V5 ~~  V5   0.631 0.070  9.036      0
13  V6 ~~  V6   0.689 0.072  9.609      0
14  F1 ~~  F1   1.000    NA     NA     NA
```

lavaan fit statistics

```
lavaan (0.5-17) converged normally after  16 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Test Statistic                3.986
  Degrees of freedom                                 8
  P-value (Chi-square)                           0.858

Model test baseline model:

  Minimum Function Test Statistic              393.670
  Degrees of freedom                                15
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.020

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1906.434
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                         13
  Akaike (AIC)                                3838.868
  Bayesian (BIC)                              3884.647
  Sample-size adjusted Bayesian (BIC)         3843.436
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.040
  P-value RMSEA <= 0.05                          0.972

Standardized Root Mean Square Residual:

  SRMR                                           0.017

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1                0.840    0.060   13.921    0.000
    V2                0.703    0.065   10.756    0.000
    V3                0.568    0.063    9.000    0.000
  F2 =~
    V4                0.791    0.063   12.599    0.000
    V5                0.617    0.067    9.142    0.000
    V6                0.531    0.064    8.348    0.000

Covariances:
  F1 ~~
    F2                0.602    0.061    9.887    0.000
```

Create two new models with some equality constraints

The first sets the loadings for 1-3 equal to those of 4-6, the second says they are all equal

R code

```
mod2fa <- 'F1 =~ a*V1 + b*V2 + c*V3
                    F2 =~ a*V4 + b*V5 + c*V6
                    F1 ~~F2'
 fit2a <- sem(mod2fa,data=x.df,std.lv=TRUE)
 summary(fit2a,fit.measures=TRUE)

mod2fe <- 'F1 =~ a*V1 + a*V2 + a*V3
                    F2 =~ a*V4 + a*V5 + a*V6
                    F1 ~~F2'
 fit2e <- sem(mod2fe,data=x.df,std.lv=TRUE)
 summary(fit2e,fit.measures=TRUE)
```

Equal across time

```
lavaan (0.5-17) converged normally after  15 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Test Statistic                5.338
  Degrees of freedom                                11
  P-value (Chi-square)                           0.914

Model test baseline model:

  Minimum Function Test Statistic              393.670
  Degrees of freedom                                15
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.020

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1907.110
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                         10
  Akaike (AIC)                                3834.220
  Bayesian (BIC)                              3869.435
  Sample-size adjusted Bayesian (BIC)         3837.734

Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.025
  P-value RMSEA <= 0.05                          0.991

Standardized Root Mean Square Residual:

  SRMR                                           0.035

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.817    0.046   17.711    0.000
    V2        (b)     0.662    0.049   13.630    0.000
    V3        (c)     0.549    0.046   11.949    0.000
  F2 =~
    V4        (a)     0.817    0.046   17.711    0.000
    V5        (b)     0.662    0.049   13.630    0.000
    V6        (c)     0.549    0.046   11.949    0.000

Covariances:
  F1 ~~
    F2                0.599    0.061    9.861    0.000
```

All loadings equal

```
> summary(fit2e,fit.measures=TRUE)
llavaan (0.5-17) converged normally after  12 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Test Statistic               26.140
  Degrees of freedom                                13
  P-value (Chi-square)                           0.016

Model test baseline model:

  Minimum Function Test Statistic              393.670
  Degrees of freedom                                15
  P-value                                        0.000

User model versus baseline model:

  Comparative Fit Index (CFI)                    0.965
  Tucker-Lewis Index (TLI)                       0.960

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1917.511
  Loglikelihood unrestricted model (H1)      -1904.441

  Number of free parameters                          8
  Akaike (AIC)                                3851.022
  Bayesian (BIC)                              3879.194
  Sample-size adjusted Bayesian (BIC)         3853.833
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.064
  90 Percent Confidence Interval          0.026  0.099
  P-value RMSEA <= 0.05                          0.235

Standardized Root Mean Square Residual:

  SRMR                                           0.084

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.686    0.033   20.964    0.000
    V2        (a)     0.686    0.033   20.964    0.000
    V3        (a)     0.686    0.033   20.964    0.000
  F2 =~
    V4        (a)     0.686    0.033   20.964    0.000
    V5        (a)     0.686    0.033   20.964    0.000
    V6        (a)     0.686    0.033   20.964    0.000

Covariances:
  F1 ~~
    F2                0.631    0.062   10.115    0.000
```

Do the models differ?

These are nested models, and we can compare their χ^2 values.

R code

```
anova(fit,fit2a)
anova(fit2a,2e)
```

```
Chi Square Difference Test

      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit    8 3838.9 3884.6 3.9861
fit2a 11 3834.2 3869.4 5.3380     1.3519       3     0.7169

hi Square Difference Test

      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit2a 11 3834.2 3869.4  5.338
fit2e 13 3851.0 3879.2 26.140     20.802       2   3.04e-05 ***
```

Measurement Invariance: Does a test measure the same thing

1. Across groups
   - Different schools
   - Different groups (e.g., ethnicity, age, gender)
2. Across time
   - Is todays' measure the same as next year's measure?
3. Types of invariance
   - Configural: Are the arrows the same
   - Weak invariance: Are the loadings the same across groups
   - Strong invariance: Loadings and intercepts are equal across groups
   - Super strong: Loadings, intercepts and means are equal across groups

Consider the Holzinger Swineford data set

1. 9 ability measures from two schools
   - 145 from Grant-White
   - 156 from Pasteur
2. Are the factor structures the same across schools
   - Although lavaan does this in one call, lets do it part by part
   - over all factor structure
   - factors with schools
   - constrain factors to have the same loadings, etc.

First, some descriptive statistics

describe(HolzingerSwineford1939,skew=FALSE)

	var	n	mean	sd	median	trimmed	mad	min	max	range	se
id	1	301	176.55	105.94	163.00	176.78	140.85	1.00	351.00	350.00	6.11
sex	2	301	1.51	0.50	2.00	1.52	0.00	1.00	2.00	1.00	0.03
ageyr	3	301	13.00	1.05	13.00	12.89	1.48	11.00	16.00	5.00	0.06
agemo	4	301	5.38	3.45	5.00	5.32	4.45	0.00	11.00	11.00	0.20
school*	5	301	1.52	0.50	2.00	1.52	0.00	1.00	2.00	1.00	0.03
grade	6	300	7.48	0.50	7.00	7.47	0.00	7.00	8.00	1.00	0.03
x1	7	301	4.94	1.17	5.00	4.96	1.24	0.67	8.50	7.83	0.07
x2	8	301	6.09	1.18	6.00	6.02	1.11	2.25	9.25	7.00	0.07
x3	9	301	2.25	1.13	2.12	2.20	1.30	0.25	4.50	4.25	0.07
x4	10	301	3.06	1.16	3.00	3.02	0.99	0.00	6.33	6.33	0.07
x5	11	301	4.34	1.29	4.50	4.40	1.48	1.00	7.00	6.00	0.07
x6	12	301	2.19	1.10	2.00	2.09	1.06	0.14	6.14	6.00	0.06
x7	13	301	4.19	1.09	4.09	4.16	1.10	1.30	7.43	6.13	0.06
x8	14	301	5.53	1.01	5.50	5.49	0.96	3.05	10.00	6.95	0.06
x9	15	301	5.37	1.01	5.42	5.37	0.99	2.78	9.25	6.47	0.06

describeBy each group

```
> describeBy(HolzingerSwineford1939,group=HolzingerSwineford1939$school,skew=FAL
group: Grant-White
      var   n  mean   sd median trimmed  mad   min   max range   se
sex     2 145  1.50 0.50   2.00    1.50 0.00  1.00  2.00  1.00 0.04
ageyr   3 145 12.72 0.97  13.00   12.67 1.48 11.00 16.00  5.00 0.08
...
grade   6 144  7.45 0.50   7.00    7.44 0.00  7.00  8.00  1.00 0.04
x1      7 145  4.93 1.15   5.00    4.96 1.24  1.83  8.50  6.67 0.10
x2      8 145  6.20 1.11   6.25    6.14 1.11  2.25  9.25  7.00 0.09
...
x8     14 145  5.49 1.05   5.50    5.45 0.89  3.05 10.00  6.95 0.09
x9     15 145  5.33 1.03   5.31    5.33 1.15  3.11  9.25  6.14 0.09
----------------------------------------------------------------------------
group: Pasteur
      var   n  mean   sd median trimmed  mad   min   max range   se
sex     2 156  1.53 0.50   2.00    1.53 0.00  1.00  2.00  1.00 0.04
ageyr   3 156 13.25 1.06  13.00   13.15 1.48 12.00 16.00  4.00 0.09
...
grade   6 156  7.50 0.50   7.50    7.50 0.74  7.00  8.00  1.00 0.04
x1      7 156  4.94 1.19   5.00    4.97 1.24  0.67  7.50  6.83 0.09
x2      8 156  5.98 1.23   5.75    5.89 1.11  3.50  9.25  5.75 0.10
...
```

EFA for both groups

by(HolzingerSwineford1939[,7:15],HolzingerSwineford1939[,5],fa,nfactors=

```
HolzingerSwineford1939[, 5]: Grant-White
Factor Analysis using method =  minres
Call: FUN(r = data[x, , drop = FALSE], nfactors = 3)
Standardized loadings (pattern matrix) based upon corr
     MR1   MR2   MR3   h2   u2
x1  0.09  0.06  0.64 0.50 0.50
x2  0.02 -0.03  0.51 0.26 0.74
x3  0.11 -0.03  0.64 0.47 0.53
x4  0.86 -0.04  0.04 0.76 0.24
x5  0.82  0.09 -0.03 0.70 0.30
x6  0.81 -0.04  0.05 0.68 0.32
x7  0.14  0.78 -0.19 0.60 0.40
x8 -0.11  0.79  0.18 0.69 0.31
x9  0.08  0.46  0.40 0.54 0.46

                       MR1  MR2  MR3
SS loadings           2.23 1.53 1.44
Proportion Var        0.25 0.17 0.16
Cumulative Var        0.25 0.42 0.58
Proportion Explained  0.43 0.29 0.28
Cumulative Proportion 0.43 0.72 1.00

 With factor correlations of
     MR1  MR2  MR3
MR1 1.00 0.25 0.41
MR2 0.25 1.00 0.31
MR3 0.41 0.31 1.00
```

```
HolzingerSwineford1939[, 5]: Pasteur
Factor Analysis using method =  minres
Call: FUN(r = data[x, , drop = FALSE], nfactors = 3)
Standardized loadings (pattern matrix) based upon corr
     MR1   MR2   MR3   h2   u2
x1  0.27  0.59  0.00 0.51 0.49
x2  0.03  0.49 -0.16 0.25 0.75
x3 -0.08  0.73  0.01 0.50 0.50
x4  0.80  0.02  0.06 0.68 0.32
x5  0.92 -0.07 -0.06 0.79 0.21
x6  0.78  0.13  0.06 0.70 0.30
x7  0.06 -0.14  0.71 0.52 0.48
x8 -0.02  0.13  0.60 0.39 0.61
x9 -0.02  0.37  0.40 0.34 0.66

                       MR1  MR2  MR3
SS loadings           2.22 1.36 1.10
Proportion Var        0.25 0.15 0.12
Cumulative Var        0.25 0.40 0.52
Proportion Explained  0.48 0.29 0.23
Cumulative Proportion 0.48 0.77 1.00

 With factor correlations of
     MR1  MR2  MR3
MR1 1.00 0.27 0.26
MR2 0.27 1.00 0.14
MR3 0.26 0.14 1.00
```

How similar are the solutions: factor congruence

Factor congruence is the cosine of the angle between two vectors:

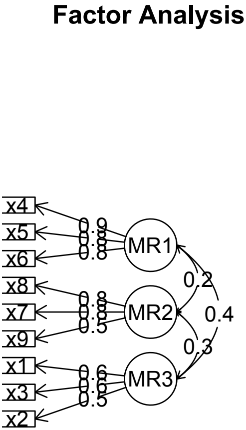
$$Congruence = (diag(X'X))^{-.5} Y'X(diag((Y'Y))^{-.5}$$

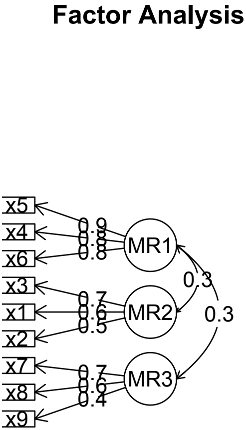
```
> f3.pasteur <- fa(HolzingerSwineford1939[1:156,7:15],3)
> f3.grant <- fa(HolzingerSwineford1939[157:301,7:15],3)
>factor.congruence(f3.pasteur,f3.grant)
```

```
     MR1  MR2  MR3
MR1 0.97 0.03 0.10
MR2 0.12 0.11 0.99
MR3 0.08 0.97 0.05
```

```
  cross <- t(y) %*% x
  sumsx <- sqrt(1/diag(t(x) %*% x))
   sumsy <- sqrt(1/diag(t(y) %*% y))
```

Do they look alike?





Test if the model fits the combined data

```
HS.model <- ' visual  =~ x1 + x2 + x3
              textual =~ x4 + x5 + x6
              speed   =~ x7 + x8 + x9 '

 fit <- cfa(HS.model, data=HolzingerSwineford1939,std.lv=TRUE)
summary(fit, fit.measures=TRUE)

lavaan (0.4-14) converged normally after 41 iterations

  Number of observations                           301

  Estimator                                         ML
  Minimum Function Chi-square                   85.306
  Degrees of freedom                                24
  P-value                                        0.000

Chi-square test baseline model:

  Minimum Function Chi-square                  918.852
  Degrees of freedom                                36
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.931
  Tucker-Lewis Index (TLI)                       0.896

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -3737.745
  Loglikelihood unrestricted model (H1)      -3695.092
```

With values of

```
Number of free parameters                          21
  Akaike (AIC)                                 7517.490
  Bayesian (BIC)                               7595.339
  Sample-size adjusted Bayesian (BIC)          7528.739

Root Mean Square Error of Approximation:

  RMSEA                                           0.092
  90 Percent Confidence Interval          0.071   0.114
  P-value RMSEA <= 0.05                           0.001

Standardized Root Mean Square Residual:

  SRMR                                            0.065

Parameter estimates:

  Information                                  Expected
  Standard Errors                              Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                0.900    0.081   11.127    0.000
    x2                0.498    0.077    6.429    0.000
    x3                0.656    0.074    8.817    0.000
  textual =~
    x4                0.990    0.057   17.474    0.000
    x5                1.102    0.063   17.576    0.000
    x6                0.917    0.054   17.082    0.000
  speed =~
    x7                0.619    0.070    8.903    0.000
```

Now do it for both groups one analysis

```
> fit2 <- cfa(HW.model, data=HolzingerSwineford1939, group="school",std.lv=TRUE)
> summary(fit2)

  Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML
  Minimum Function Chi-square                  115.851
  Degrees of freedom                                48
  P-value                                        0.000

Chi-square for each group:

  Pasteur                                       64.309
  Grant-White                                   51.542

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard
```

Results for Pasteur

```
Group 1 [Pasteur]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                1.047    0.132    7.934    0.000
    x2                0.412    0.110    3.753    0.000
    x3                0.597    0.108    5.525    0.000
  textual =~
    x4                0.946    0.079   11.927    0.000
    x5                1.119    0.089   12.604    0.000
    x6                0.827    0.068   12.230    0.000
  speed =~
    x7                0.591    0.106    5.557    0.000
    x8                0.665    0.102    6.531    0.000
    x9                0.545    0.097    5.596    0.000

Covariances:
  visual ~~
    textual           0.484    0.086    5.600    0.000
    speed             0.299    0.109    2.755    0.006
  textual ~~
    speed             0.325    0.100    3.256    0.001

Intercepts:
    x1                4.941    0.095   52.249    0.000
    x2                5.984    0.098   60.949    0.000
    x3                2.487    0.093   26.778    0.000
    x4                2.823    0.092   30.689    0.000
    x5                3.995    0.105   38.183    0.000
    x6                1.922    0.079   24.321    0.000
    x7                4.432    0.087   51.181    0.000
```

Results for Grant-White

```
Latent variables:
  visual =~
    x1              0.777    0.103    7.525    0.000
    x2              0.572    0.101    5.642    0.000
    x3              0.719    0.093    7.711    0.000
  textual =~
    x4              0.971    0.079   12.355    0.000
    x5              0.961    0.083   11.630    0.000
    x6              0.935    0.081   11.572    0.000
  speed =~
    x7              0.679    0.087    7.819    0.000
    x8              0.833    0.087    9.568    0.000
    x9              0.719    0.086    8.357    0.000

Covariances:
  visual ~~
    textual         0.541    0.085    6.355    0.000
    speed           0.523    0.094    5.562    0.000
  textual ~~
    speed           0.336    0.091    3.674    0.000

Intercepts:
    x1              4.930    0.095   51.696    0.000
    x2              6.200    0.092   67.416    0.000
    x3              1.996    0.086   23.195    0.000
    x4              3.317    0.093   35.625    0.000
    x5              4.712    0.096   48.986    0.000
    x6              2.469    0.094   26.277    0.000
    x7              3.921    0.086   45.819    0.000
    x8              5.488    0.087   63.174    0.000
    x9              5.327    0.085   62.571    0.000
    visual          0.000
```

Constrain the loadings to be the same

```
> fit2e <- cfa(HW.model, data=HolzingerSwineford1939, group="school",std.lv=TRUE,group.equal="loadings")
> summary(fit2e)

lavaan (0.4-14) converged normally after 30 iterations

  Number of observations per group
  Pasteur                                            156
  Grant-White                                        145

  Estimator                                           ML
  Minimum Function Chi-square                    127.834
  Degrees of freedom                                  57
  P-value                                          0.000

Chi-square for each group:

  Pasteur                                         71.064
  Grant-White                                     56.770

Parameter estimates:

  Information                                   Expected
  Standard Errors                               Standard
```

With parameter values of

```
Group 1 [Pasteur]:

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                0.866    0.078   11.149    0.000
    x2                0.523    0.076    6.916    0.000
    x3                0.683    0.071    9.689    0.000
  textual =~
    x4                0.954    0.056   17.002    0.000
    x5                1.033    0.061   17.012    0.000
    x6                0.870    0.052   16.750    0.000
  speed =~
    x7                0.630    0.066    9.500    0.000
    x8                0.752    0.065   11.586    0.000
    x9                0.650    0.064   10.205    0.000

Covariances:
  visual ~~
    textual           0.485    0.087    5.555    0.000
    speed             0.341    0.109    3.126    0.002
  textual ~~
    speed             0.336    0.094    3.590    0.000

Intercepts:
    x1                4.941    0.092   53.661    0.000
    x2                5.984    0.099   60.420    0.000
    x3                2.487    0.093   26.734    0.000
    x4                2.823    0.093   30.400    0.000
    x5                3.995    0.100   39.756    0.000
    x6                1.922    0.081   23.732    0.000
```

Now, compare the fit of the two group model to the one with equal parameters

```
>  anova(fit2,fit2e)

Chi Square Difference Test

      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit2  48 7484.4 7706.8 115.85
fit2e 57 7478.4 7667.4 127.83     11.982       9     0.2143
```

But, another way to specify the fit2e model – without making the latent variables standardized

fit2e <- cfa(HW.model, data=HolzingerSwineford1939, group="school",group.equal=c("loadings"))

```
 Number of observations per group
  Pasteur                                          156
  Grant-White                                      145

  Estimator                                         ML
  Minimum Function Chi-square                  124.044
  Degrees of freedom                                54
  P-value                                        0.000

Chi-square for each group:
  Pasteur                                       68.825
  Grant-White                                   55.219
Parameter estimates:
  Information                                 Expected
  Standard Errors                             Standard

Group 1 [Pasteur]:
                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  visual =~
    x1                1.000
    x2                0.599    0.100    5.979    0.000
    x3                0.784    0.108    7.267    0.000
  textual =~
    x4                1.000
    x5                1.083    0.067   16.049    0.000
    x6                0.912    0.058   15.785    0.000
  speed =~
    x7                1.000
```

Test fit by comparing models

```
> anova(fit2,fit2e)

Chi Square Difference Test

      Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit2  48 7484.4 7706.8 115.85
fit2e 54 7480.6 7680.8 124.04     8.1922       6     0.2244
```

Continue this logic, of successive tests with more constraints, but do it automatically

```
 mi <- measurementInvariance(HW.model, data=HolzingerSwineford1939, group="school")
Measurement invariance tests:
Model 1: configural invariance:
   chisq        df    pvalue       cfi     rmsea       bic
 115.851    48.000     0.000     0.923     0.097  7706.822
Model 2: weak invariance (equal loadings):
   chisq        df    pvalue       cfi     rmsea       bic
 124.044    54.000     0.000     0.921     0.093  7680.771
[Model 1 versus model 2]
  delta.chisq      delta.df delta.p.value     delta.cfi
        8.192         6.000         0.224         0.002
Model 3: strong invariance (equal loadings + intercepts):
   chisq        df    pvalue       cfi     rmsea       bic
 164.103    60.000     0.000     0.882     0.107  7686.588
[Model 1 versus model 3]
  delta.chisq      delta.df delta.p.value     delta.cfi
       48.251        12.000         0.000         0.041
[Model 2 versus model 3]
  delta.chisq      delta.df delta.p.value     delta.cfi
       40.059         6.000         0.000         0.038
Model 4: equal loadings + intercepts + means:
   chisq        df    pvalue       cfi     rmsea       bic
 204.605    63.000     0.000     0.840     0.122  7709.969
[Model 1 versus model 4]
  delta.chisq      delta.df delta.p.value     delta.cfi
       88.754        15.000         0.000         0.083
[Model 3 versus model 4]
  delta.chisq      delta.df delta.p.value     delta.cfi
       40.502         3.000         0.000         0.042
>
```

Create a data set with non-invariant factor loadings

```
> set.seed(42)
 > fx <- matrix(c(.8,.7,.6,rep(0,6),.6,.7,.8),ncol=2)
> fx
     [,1] [,2]
[1,]  0.8  0.0
[2,]  0.7  0.0
[3,]  0.6  0.0
[4,]  0.0  0.6
[5,]  0.0  0.7
[6,]  0.0  0.8
> Phi <- matrix(c(1,.6,.6,1),ncol=2)
> Phi
> set.seed(42)
> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
>  x <- x.model$observed
>  structure.diagram(fx,Phi,lr=FALSE,e.size=.3,main="A basic two time model")
>  describe(x,skew=FALSE)
```

```
   var   n  mean   sd median trimmed  mad   min  max range   se
V1   1 250  0.02 0.99   0.01    0.03 1.02 -2.64 2.76  5.40 0.06
V2   2 250 -0.02 0.96  -0.01   -0.03 0.94 -2.66 3.12  5.78 0.06
V3   3 250  0.02 0.97  -0.06    0.01 0.95 -2.74 2.41  5.15 0.06
V4   4 250  0.61 0.99   0.59    0.65 0.99 -3.26 3.15  6.41 0.06
```

One factor model

```
> f1n <- fa(x)
> f1n


Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix) based upon correlation matrix
    MR1   h2   u2
V1 0.59 0.35 0.65
V2 0.56 0.31 0.69
V3 0.48 0.23 0.77
V4 0.54 0.29 0.71
V5 0.69 0.48 0.52
V6 0.72 0.52 0.48

                MR1
SS loadings    2.18
Proportion Var 0.36

Test of the hypothesis that 1 factor is sufficient.

The degrees of freedom for the null model are  15
         and the objective function was  1.58
          with Chi Square of  389.85
The degrees of freedom for the model are 9
           and the objective function was  0.33
```

```
The root mean square of the residuals (RMSR) is  0.07
The df corrected root mean square of the residuals is
The number of observations was  250  with Chi Square =

Tucker Lewis Index of factoring reliability =  0.684
RMSEA index =  0.179  and the 90 % confidence interval
BIC =  30.24
Fit based upon off diagonal values = 0.93
Measures of factor score adequacy
                                                 MR1
Correlation of scores with factors              0.89
Multiple R square of scores with factors        0.79
Minimum correlation of possible factor scores   0.57
```

Two factor model

```
> f2n <- fa(x,2)
> f2n


Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   MR2   h2   u2
V1 -0.01  0.79 0.62 0.38
V2 -0.01  0.73 0.53 0.47
V3  0.15  0.40 0.25 0.75
V4  0.51  0.07 0.30 0.70
V5  0.79 -0.03 0.60 0.40
V6  0.80  0.02 0.65 0.35

                       MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.54
MR2 0.54 1.00

Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  15
             and the objective function was  1.58
             with Chi Square of  389.85
The degrees of freedom for the model are 4
                     and the objective function was  0

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is
The number of observations was  250  with
                Chi Square =  4.7  with prob <  0.32

Tucker Lewis Index of factoring reliability =  0.993
RMSEA index =  0.028  and the 90 % confidence interval
BIC =  -17.39
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                  MR1  M

Correlation of scores with factors             0.90 0.88
Multiple R square of scores with factors       0.80 0.77
Minimum correlation of possible factor scores  0.61 0.54
```

Compare the two solutions

```
Factor Analysis using method =  minres
Call: fa(r = x)
Standardized loadings (pattern matrix)
  based upon correlation matrix
    MR1   h2   u2
V1 0.59 0.35 0.65
V2 0.56 0.31 0.69
V3 0.48 0.23 0.77
V4 0.54 0.29 0.71
V5 0.69 0.48 0.52
V6 0.72 0.52 0.48

                MR1
SS loadings    2.18
Proportion Var 0.36
```

```
Factor Analysis using method =  minres
Call: fa(r = x, nfactors = 2)
Standardized loadings (pattern matrix)
        based upon correlation matrix
     MR1   MR2   h2   u2
V1 -0.01  0.79 0.62 0.38
V2 -0.01  0.73 0.53 0.47
V3  0.15  0.40 0.25 0.75
V4  0.51  0.07 0.30 0.70
V5  0.79 -0.03 0.60 0.40
V6  0.80  0.02 0.65 0.35

                       MR1  MR2
SS loadings           1.58 1.37
Proportion Var        0.26 0.23
Cumulative Var        0.26 0.49
Proportion Explained  0.53 0.47
Cumulative Proportion 0.53 1.00

 With factor correlations of
     MR1  MR2
MR1 1.00 0.54
MR2 0.54 1.00
```

CFA 2 correlated factors

```
> y.df <- data.frame(y)
> fitn<- sem(mod2f,data=y.df,std.lv=TRUE)
> summary(fitn, fit.measures=TRUE)


lavaan (0.4-14) converged normally after 17 iterations
  Number of observations                           250
  Estimator                                         ML
  Minimum Function Chi-square                    2.552
  Degrees of freedom                                 8
  P-value                                        0.959
Chi-square test baseline model:

  Minimum Function Chi-square                  341.309
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.031

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1952.265
  Loglikelihood unrestricted model (H1)      -1950.989

  Number of free parameters                         13
  Akaike (AIC)                                3930.530
  Bayesian (BIC)                              3976.309
  Sample-size adjusted Bayesian (BIC)          3935.09
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.000
  P-value RMSEA <= 0.05                          0.994

Standardized Root Mean Square Residual:

  SRMR                                           0.014

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1                0.750    0.063   11.880    0.000
    V2                0.725    0.065   11.160    0.000
    V3                0.621    0.066    9.410    0.000
  F2 =~
    V4                0.585    0.074    7.896    0.000
    V5                0.610    0.068    8.950    0.000
    V6                0.700    0.066   10.545    0.000

Covariances:
  F1 ~~
    F2                0.607    0.067    9.067    0.000
```

Are the factors measurement invariant? Well, maybe.

```
> summary(fitn2a,fit.measures=TRUE)

lavaan (0.4-14) converged normally after 14 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                    8.093
  Degrees of freedom                                11
  P-value                                        0.705

Chi-square test baseline model:

  Minimum Function Chi-square                  341.309
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    1.000
  Tucker-Lewis Index (TLI)                       1.012

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1955.036
  Loglikelihood unrestricted model (H1)      -1950.989

  Number of free parameters                         10
  Akaike (AIC)                                3930.072
  Bayesian (BIC)                              3965.286
```

```
Root Mean Square Error of Approximation:

  RMSEA                                          0.000
  90 Percent Confidence Interval          0.000  0.051
  P-value RMSEA <= 0.05                          0.946

Standardized Root Mean Square Residual:

  SRMR                                           0.049

Parameter estimates:

  Information                                 Expected
  Standard Errors                             Standard

                   Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)     0.686    0.050   13.774    0.000
    V2        (b)     0.675    0.049   13.834    0.000
    V3        (c)     0.657    0.048   13.677    0.000
  F2 =~
    V4        (a)     0.686    0.050   13.774    0.000
    V5        (b)     0.675    0.049   13.834    0.000
    V6        (c)     0.657    0.048   13.677    0.000

Covariances:
  F1 ~~
    F2                0.610    0.067    9.089    0.000

Variances:
```

Create the original data set again

```
> fx <- matrix(c(.8,.7,.6,rep(0,6),.8,.7,.6),ncol=2)
> x.model <- sim(fx=fx,Phi=Phi,mu=c(0,1),n=250)
> x <- x.model$observed
> x.df <- data.frame(x)
```

```
> describe(x,skew=FALSE)
   var   n mean   sd median trimmed  mad   min  max range   se
V1   1 250 0.02 0.99  -0.01   -0.01 1.04 -2.38 3.26  5.64 0.06
V2   2 250 0.13 1.07   0.03    0.11 1.08 -2.53 3.65  6.19 0.07
V3   3 250 0.03 1.03   0.05    0.02 0.97 -2.68 3.12  5.80 0.07
V4   4 250 0.87 1.04   0.90    0.85 1.03 -1.63 3.61  5.24 0.07
V5   5 250 0.73 0.97   0.76    0.75 0.90 -2.36 3.29  5.65 0.06
V6   6 250 0.65 1.00   0.67    0.64 1.03 -2.58 3.17  5.75 0.06
```

Model change in the items

```
mod2fc <- 'F1 =~ a*V1 + b* V2 + c*V3
              F2 =~ a* V4 + b*V5 +c* V6
                #correlation between factors
               F2 ~  F1 '    #the regression
fit2c <- sem(mod2fc,data=x.df,meanstructure=TRUE)
summary(fit2c,fit.measures=TRUE)
```

frame

```
lavaan (0.4-14) converged normally after 23 iterations

  Number of observations                           250

  Estimator                                         ML
  Minimum Function Chi-square                   10.616
  Degrees of freedom                                10
  P-value                                        0.388

Chi-square test baseline model:

  Minimum Function Chi-square                  406.795
  Degrees of freedom                                15
  P-value                                        0.000

Full model versus baseline model:

  Comparative Fit Index (CFI)                    0.998
  Tucker-Lewis Index (TLI)                       0.998

Loglikelihood and Information Criteria:

  Loglikelihood user model (H0)              -1952.674
  Loglikelihood unrestricted model (H1)      -1947.365

  Number of free parameters                         17
  Akaike (AIC)                                3939.347
  Bayesian (BIC)                              3999.212
  Sample-size adjusted Bayesian (BIC)         3945.320
```

```
Root Mean Square Error of Approximation:

  RMSEA                                                  0.016
  90 Percent Confidence Interval             0.000       0.071
  P-value RMSEA <= 0.05                                  0.793

Standardized Root Mean Square Residual:

  SRMR                                                   0.032
Parameter estimates:

  Information                                         Expected
  Standard Errors                                     Standard

                       Estimate  Std.err  Z-value  P(>|z|)
Latent variables:
  F1 =~
    V1        (a)       1.000
    V2        (b)       0.925    0.076   12.243    0.000
    V3        (c)       0.816    0.072   11.386    0.000
  F2 =~
    V4        (a)       1.000
    V5        (b)       0.925    0.076   12.243    0.000
    V6        (c)       0.816    0.072   11.386    0.000

Regressions:
  F2 ~
    F1                  0.644    0.075    8.593    0.000

Intercepts:
```

With the intercepts

Intercepts:

V1	0.021	0.063	0.332	0.740
V2	0.135	0.066	2.033	0.042
V3	0.032	0.066	0.482	0.630
V4	0.865	0.065	13.294	0.000
V5	0.729	0.062	11.696	0.000
V6	0.649	0.063	10.369	0.000
F1	0.000			
F2	0.000			

Variances:

V1	0.383	0.060
V2	0.576	0.068
V3	0.670	0.071
V4	0.486	0.065
V5	0.481	0.061
V6	0.596	0.065
F1	0.613	0.088
F2	0.319	0.063

Try fitting a moments model

```
mod2fc <- 'F1 =~ a*V1 + b* V2 + c*V3
                F2 =~ a* V4 + b*V5 +c* V6
                means = ~ F1 + F2 + 1*V1 +1*V2+1*V3+1*V4+1*V5+1*V6
                  #correlation between factors

                F2 ~ F1 '    #the regression
fit2c <- sem(mod2fc,data=x.df,meanstructure=TRUE)
summary(fit2c,fit.measures=TRUE)
```

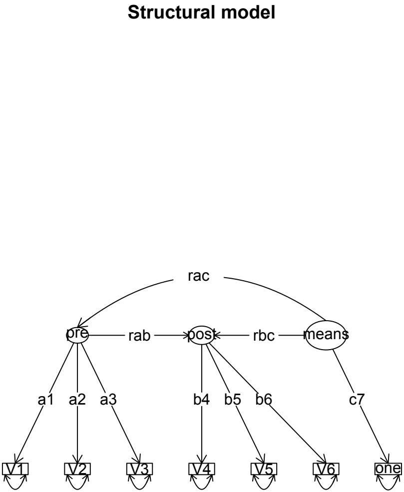
Modeling change using the moments matrix

1. McArdle (2009) Latent variable modeling of differences and changes with longitudinal data. Annual Review of Psychology, 60, 577-605
   - Use moments rather than covariances
2. Probably can be done in lavaan, but I don't know how. Can be done in sem package

Create the model to be fit in sem

```
 fxg
    pre  post means
V1  "a1" "0"  "0"
V2  "a2" "0"  "0"
V3  "a3" "0"  "0"
V4  "0"  "b4" "0"
V5  "0"  "b5" "0"
V6  "0"  "b6" "0"
one "0"  "0"  "c7"
> phi
   F1    F2  F3
F1 "1"   "0" "rac"
F2 "rab" "1" "rbc"
F3 "0"   "0" "1"

mod.mom1 <- structure.diagram(fog,phi,errors=TRUE)
```

Modeling the means in a moments matrix



the basic path model, with some editing

```
mod.mom1
      Path               Parameter Value
 [1,] "pre->V1"          "a1"      NA
 [2,] "pre->V2"          "a2"      NA
 [3,] "pre->V3"          "a3"      NA
 [4,] "post->V4"         "b4"      NA
 [5,] "post->V5"         "b5"      NA
 [6,] "post->V6"         "b6"      NA
 [7,] "means->one"       "c7"      NA
 [8,] "V1<->V1"          "x1e"     NA
 [9,] "V2<->V2"          "x2e"     NA
[10,] "V3<->V3"          "x3e"     NA
[11,] "V4<->V4"          "x4e"     NA
[12,] "V5<->V5"          "x5e"     NA
[13,] "V6<->V6"          "x6e"     NA
[14,] "one<->one"        NA    "1"      <-- edited
[15,] "pre ->post"       "rF1F2"   NA
[16,] "means->pre"       "rF3F1"   NA    <- edited
[17,] "means->post"      "rF3F2"   NA   <- edited
[18,] "pre<->pre"        NA        "1"
[19,] "post<->post"      NA        "1"
[20,] "means<->means"    NA        "1"
attr(,"class")
[1] "mod"
```

sem output

```
> sem.mom1 <- sem(mod.mom,MomMat,N=250,raw=TRUE)
> summary(sem.mom1)


Model fit to raw moment matrix.

 Model Chisquare =  12.077   Df =  12
                  Pr(>Chisq) = 0.43954
 AIC =  44.077
 AICc =  14.411
 BIC =  100.42
 CAIC =  -66.181

 Normalized Residuals
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-1.2500 -0.1150  0.0000 -0.0103  0.0972  0.9730
```

```
 Parameter Estimates
      Estimate Std Error z value Pr(>|z|)
a1    0.830111 0.069823  11.8888  1.3536e-32 V1 <--- pre
a2    0.881776 0.076506  11.5256  9.7982e-31 V2 <--- pre
a3    0.698963 0.074824   9.3415  9.5005e-21 V3 <--- pre
b4    0.664838 0.062395  10.6553  1.6468e-26 V4 <--- post
b5    0.554498 0.053510  10.3626  3.6687e-25 V5 <--- post
b6    0.510270 0.051499   9.9084  3.8265e-23 V6 <--- post
c7    1.118034 0.050000  22.3607 9.5054e-111 one <--- means
x1e   0.525613 0.078258   6.7164  1.8623e-11 V1 <--> V1
x2e   0.673405 0.093381   7.2114  5.5387e-13 V2 <--> V2
x3e   0.836504 0.090362   9.2572  2.0983e-20 V3 <--> V3
x4e   0.567286 0.081353   6.9732  3.0990e-12 V4 <--> V4
x5e   0.628041 0.073463   8.5490  1.2412e-17 V5 <--> V5
x6e   0.750522 0.080300   9.3465  9.0592e-21 V6 <--> V6
rF1F2 0.893184 0.142998   6.2461  4.2076e-10 post <--- pre
rF3F1 0.086583 0.073172   1.1833  2.3669e-01 pre <--- means
rF3F2 1.375097 0.159247   8.6350  5.8723e-18 post <--- means

 Iterations =  21
```

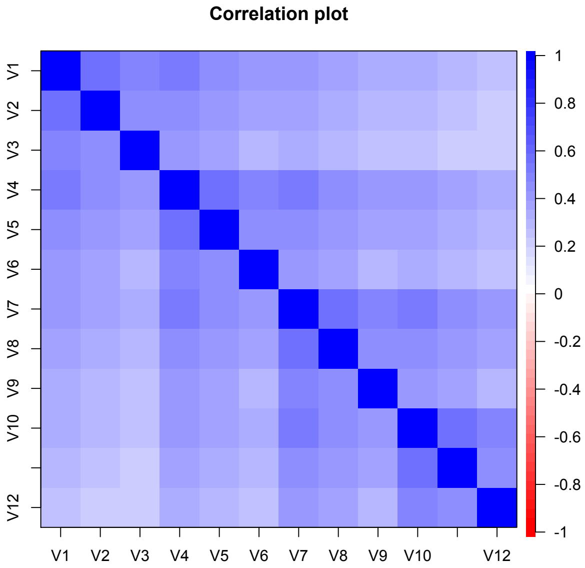
Traits and States and time

1. With just two time points, traits and states are confounded
   - Is the correlation a trait like stability
   - or does the state dissipate slowly?
2. With > 2 time points we can distinguish states and traits
   - States should have an autocorrelation component
   - Traits should be consistent across time
3. Consider the simplex structure of 4 time points
   - Clean within time factor structure
   - Simplex across time points

A factor simplex

```
simp <- sim()

 $model (Population correlation matrix)
      V1   V2   V3   V4   V5   V6   V7   V8   V9  V10  V11  V12
V1  1.00 0.56 0.48 0.51 0.45 0.38 0.41 0.36 0.31 0.33 0.29 0.25
V2  0.56 1.00 0.42 0.45 0.39 0.34 0.36 0.31 0.27 0.29 0.25 0.22
V3  0.48 0.42 1.00 0.38 0.34 0.29 0.31 0.27 0.23 0.25 0.22 0.18
V4  0.51 0.45 0.38 1.00 0.56 0.48 0.51 0.45 0.38 0.41 0.36 0.31
V5  0.45 0.39 0.34 0.56 1.00 0.42 0.45 0.39 0.34 0.36 0.31 0.27
V6  0.38 0.34 0.29 0.48 0.42 1.00 0.38 0.34 0.29 0.31 0.27 0.23
V7  0.41 0.36 0.31 0.51 0.45 0.38 1.00 0.56 0.48 0.51 0.45 0.38
V8  0.36 0.31 0.27 0.45 0.39 0.34 0.56 1.00 0.42 0.45 0.39 0.34
V9  0.31 0.27 0.23 0.38 0.34 0.29 0.48 0.42 1.00 0.38 0.34 0.29
V10 0.33 0.29 0.25 0.41 0.36 0.31 0.51 0.45 0.38 1.00 0.56 0.48
V11 0.29 0.25 0.22 0.36 0.31 0.27 0.45 0.39 0.34 0.56 1.00 0.42
V12 0.25 0.22 0.18 0.31 0.27 0.23 0.38 0.34 0.29 0.48 0.42 1.00
```

A simplex


Correlation plot
V1
V2
V3
V4
V5
V6
V7
V8
V9
V10
V12
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

Factor structure of a simplex

```
> fsimp <- fa(simp$model)
> fsimp

Factor Analysis using method =  minres
Call: fa(r = simp$model)
Standardized loadings (pattern matrix) based upon correlation matrix
     MR1   h2   u2
V1  0.64 0.41 0.59
V2  0.57 0.33 0.67
V3  0.49 0.24 0.76
V4  0.73 0.54 0.46
V5  0.65 0.42 0.58
V6  0.56 0.31 0.69
V7  0.73 0.54 0.46
V8  0.65 0.42 0.58
V9  0.56 0.31 0.69
V10 0.64 0.41 0.59
V11 0.57 0.33 0.67
V12 0.49 0.24 0.76

                MR1
SS loadings    4.50
Proportion Var 0.38
```

Factors over time

```
> fsimp4 <- fa(simp$model,4)
> fsimp4

Factor Analysis using method =  minres
Call: fa(r = simp$model, nfactors = 4)
Standardized loadings (pattern matrix)
            based upon correlation matrix
    MR3 MR1 MR2 MR4   h2   u2
V1  0.8 0.0 0.0 0.0 0.64 0.36
V2  0.7 0.0 0.0 0.0 0.49 0.51
V3  0.6 0.0 0.0 0.0 0.36 0.64
V4  0.0 0.0 0.0 0.8 0.64 0.36
V5  0.0 0.0 0.0 0.7 0.49 0.51
V6  0.0 0.0 0.0 0.6 0.36 0.64
V7  0.0 0.8 0.0 0.0 0.64 0.36
V8  0.0 0.7 0.0 0.0 0.49 0.51
V9  0.0 0.6 0.0 0.0 0.36 0.64
V10 0.0 0.0 0.8 0.0 0.64 0.36
V11 0.0 0.0 0.7 0.0 0.49 0.51
V12 0.0 0.0 0.6 0.0 0.36 0.64

                       MR3  MR1  MR2  MR4
SS loadings           1.49 1.49 1.49 1.49
Proportion Var        0.12 0.12 0.12 0.12
Cumulative Var        0.12 0.25 0.37 0.50
Proportion Explained  0.25 0.25 0.25 0.25
Cumulative Proportion 0.25 0.50 0.75 1.00

 With factor correlations of
     MR3  MR1  MR2  MR4
MR3 1.00 0.64 0.51 0.80
MR1 0.64 1.00 0.80 0.80
MR2 0.51 0.80 1.00 0.64
MR4 0.80 0.80 0.64 1.00
```

Bollen, K. A. (2002). *Latent variables in psychology and the social sciences*. US: Annual Reviews.

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Kerckhoff, A. C. (1974). *Ambition and Attainment: A Study of Four Samples of American Boys.* Washington, D. C.: American Sociological Association.