

psych: a general purpose toolkit for personality and psychological research

Presented at the Southwestern Psychological Association
meeting,
San Antonio, Texas April, 2017

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slides: <http://personality-project.org/r/tutorials/swpa/swpa.pdf>

Outline

Preliminaries

Installing psych and listing all its objects

Getting and cleaning data

Descriptive and inferential statistics

Graphical displays

Some inferential statistics – testing correlations

Scores and Reliability

Reliability

Scoring Scales

Alternative estimates of internal consistency: α, β, ω_h

Estimating $\omega_{hierarchical}$ and ω_{total} using omega

Exploratory Factor Analysis

How many factors are in the mood data

Factor extraction and graphical displays

Hierarchical Analysis

Graphical displays of hierarchical analysis

Mediation, moderation, and set correlation

Multiple Regression



Installing the psych package ($\geq 1.7.3$)

R code

```
#if you have not already done so, you first install the package
install.packages("psych",dependencies=TRUE)
```

```
library(psych) #you need to do this every time you start R
```

```
#or automate the library(psych) call
#by creating and saving a function
```

```
.First <- function() {library(psych)}
quit() #with save option
```

```
#start R and psych will be automatically loaded
sessionInfo() #will tell you what version you are using
```

Good morning Bill.

Are you ready to have some fun?

```
> sessionInfo()
```

```
R version 3.3.3 Patched (2017-03-06 r72351)
```

```
Platform: x86_64-apple-darwin13.4.0 (64-bit)
```

```
Running under: macOS Sierra 10.12.3
```

```
locale:
```

```
[1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
```



Show all the functions in the psych package objects ("package:psych")

```
[1] "%+%"                "ability"                "affect"                "all.income"
[6] "anova.psych"        "autoR"                  "Bechtoldt"            "Bechtoldt.1"
...
[81] "cushny"              "d2r"                    "densityBy"            "describe"
[86] "describeBy"         "describeData"          "df2latex"             "dfOrder"
...
[111] "error.bars"         "error.bars.by"         "error.bars.tab"      "error.cross"
[116] "errorCircles"      "esem"                  "esem.diagram"        "fa"
...
[131] "fa.plot"            "fa.poly"               "fa.rgraph"           "fa.sort"
[136] "fa2irt"             "fa2latex"              "faBy"                "fac"
...
[176] "ICC"                "ICC2latex"             "iclust"              "ICLUST"
[181] "iclust.diagram"    "ICLUST.graph"         "ICLUST.rgraph"      "iclust.sort"
...
[201] "irt.fa"             "irt.item.diff.rasch"   "irt.person.rasch"    "irt.respons"
[206] "irt.stats.like"    "irt.tau"              "irt2latex"           "isCorrelati"
[211] "item.lookup"       "item.sim"             "kaiser"              "keys.lookup"
...
[251] "omega"              "omega.diagram"        "omega.graph"         "omega2latex"
[256] "omegah"            "omegaSem"             "outlier"            "p.rep"
...
[301] "read.clipboard"    "read.clipboard.csv"   "read.clipboard.fwf"  "read.clipbo"
[306] "read.clipboard.upper" "read.file"           "read.file.csv"      "read.https"
...
[326] "score.alpha"       "score.irt"            "score.irt.2"         "score.irt.p"
[331] "score.multiple.choice" "scoreFast"          "scoreIrt"           "scoreIrt.1p"
[336] "scoreItems"        "scoreOverlap"        "scree"              "scrub"
...
[356] "sim.multi"         "sim.multilevel"       "sim.npl"            "sim.npn"
...
```

Get your data: using `read.file` or `read.clipboard`

From a website: define the file name

R code

```
fn <- "http://personality-project.org/r/datasets/Maps.mixx.ms1.epi.bf.txt"
fn #show it to check
[1] "http://personality-project.org/r/datasets/Maps.mixx.ms1.epi.bf.txt"
mydata <- read.file(fn,header=TRUE)
```

From a local file: find the file using `read.file`

R code

```
> my.data <- read.file() #will open a search window, read the file
#depending upon the suffix, will read .sav, .csv, .txt, .rds, .rDa, et
```

From the clipboard: (first, go to the remote site, copy to the clipboard and then use the `read.clipboard` function).

R code

```
mydata <- read.clipboard() #or
mydata <- read.clipboard.tab() #if an excel file
my.data <- read.clipboard.csv() #if a tab delimited file
```

Checking the data using describe

```
> dim(mydata)
```

```
[1] 231 86
```

```
> describe(mydata)
```

| | vars | n | mean | sd | median | trimmed | mad | min | max | range | skew | kurtosis | se |
|-----------|------|-----|-------|-------|--------|---------|-------|-----|-----|-------|-------|----------|------|
| id | 1 | 231 | 66.82 | 45.13 | 58 | 64.14 | 50.41 | 1 | 160 | 159 | 0.45 | -0.96 | 2.97 |
| delighted | 2 | 231 | 0.82 | 1.05 | 1 | 0.68 | 1.48 | 0 | 9 | 9 | 2.46 | 14.15 | 0.07 |
| sociable | 3 | 231 | 1.32 | 0.96 | 1 | 1.28 | 1.48 | 0 | 3 | 3 | 0.06 | -1.03 | 0.06 |
| jittery | 4 | 231 | 0.55 | 0.78 | 0 | 0.39 | 0.00 | 0 | 3 | 3 | 1.37 | 1.23 | 0.05 |
| hostile | 5 | 231 | 0.35 | 0.85 | 0 | 0.17 | 0.00 | 0 | 9 | 9 | 5.34 | 45.21 | 0.06 |
| sluggish | 6 | 231 | 1.21 | 0.96 | 1 | 1.14 | 1.48 | 0 | 3 | 3 | 0.47 | -0.70 | 0.06 |
| depressed | 7 | 231 | 0.56 | 0.83 | 0 | 0.39 | 0.00 | 0 | 3 | 3 | 1.45 | 1.37 | 0.05 |
| ... | | | | | | | | | | | | | |
| ashamed | 71 | 231 | 0.32 | 1.15 | 0 | 0.06 | 0.00 | 0 | 9 | 9 | 5.92 | 40.25 | 0.08 |
| anxious | 72 | 231 | 0.75 | 1.26 | 0 | 0.53 | 0.00 | 0 | 9 | 9 | 3.85 | 21.39 | 0.08 |
| idle | 73 | 231 | 0.98 | 1.15 | 1 | 0.83 | 1.48 | 0 | 9 | 9 | 3.11 | 18.20 | 0.08 |
| epiE | 74 | 231 | 13.33 | 4.14 | 14 | 13.49 | 4.45 | 1 | 22 | 21 | -0.33 | -0.06 | 0.27 |
| epiS | 75 | 231 | 7.58 | 2.69 | 8 | 7.77 | 2.97 | 0 | 13 | 13 | -0.57 | -0.02 | 0.18 |
| epiImp | 76 | 231 | 4.37 | 1.88 | 4 | 4.36 | 1.48 | 0 | 9 | 9 | 0.06 | -0.62 | 0.12 |
| epilie | 77 | 231 | 2.38 | 1.50 | 2 | 2.27 | 1.48 | 0 | 7 | 7 | 0.66 | 0.24 | 0.10 |
| ... | | | | | | | | | | | | | |
| traitanx | 85 | 231 | 39.01 | 9.52 | 38 | 38.36 | 8.90 | 22 | 71 | 49 | 0.67 | 0.47 | 0.63 |
| stateanx | 86 | 231 | 39.85 | 11.48 | 38 | 38.92 | 10.38 | 21 | 79 | 58 | 0.72 | -0.01 | 0.76 |



Cleaning the data using scrub

We want to change 9s in variables 2 - 73 into NA

```
> cleaned <- scrub(mydata, where=2:73, isvalue=9, newvalue=NA)
> describe(cleaned)
```

| | vars | n | mean | sd | median | trimmed | mad | min | max | range | skew | kurtosis | se |
|-----------|------|-----|-------|-------|--------|---------|-------|-----|-----|-------|-------|----------|------|
| id | 1 | 231 | 66.82 | 45.13 | 58.0 | 64.14 | 50.41 | 1 | 160 | 159 | 0.45 | -0.96 | 2.97 |
| delighted | 2 | 230 | 0.78 | 0.90 | 1.0 | 0.67 | 1.48 | 0 | 3 | 3 | 0.79 | -0.52 | 0.06 |
| sociable | 3 | 231 | 1.32 | 0.96 | 1.0 | 1.28 | 1.48 | 0 | 3 | 3 | 0.06 | -1.03 | 0.06 |
| jittery | 4 | 231 | 0.55 | 0.78 | 0.0 | 0.39 | 0.00 | 0 | 3 | 3 | 1.37 | 1.23 | 0.05 |
| hostile | 5 | 230 | 0.31 | 0.63 | 0.0 | 0.17 | 0.00 | 0 | 3 | 3 | 2.12 | 4.19 | 0.04 |
| sluggish | 6 | 231 | 1.21 | 0.96 | 1.0 | 1.14 | 1.48 | 0 | 3 | 3 | 0.47 | -0.70 | 0.06 |
| depressed | 7 | 231 | 0.56 | 0.83 | 0.0 | 0.39 | 0.00 | 0 | 3 | 3 | 1.45 | 1.37 | 0.05 |
| ... | | | | | | | | | | | | | |
| ashamed | 71 | 228 | 0.21 | 0.57 | 0.0 | 0.05 | 0.00 | 0 | 3 | 3 | 3.00 | 8.95 | 0.04 |
| anxious | 72 | 228 | 0.64 | 0.84 | 0.0 | 0.51 | 0.00 | 0 | 3 | 3 | 1.11 | 0.32 | 0.06 |
| idle | 73 | 229 | 0.91 | 0.88 | 1.0 | 0.82 | 1.48 | 0 | 3 | 3 | 0.64 | -0.42 | 0.06 |
| epiE | 74 | 231 | 13.33 | 4.14 | 14.0 | 13.49 | 4.45 | 1 | 22 | 21 | -0.33 | -0.06 | 0.27 |
| epiS | 75 | 231 | 7.58 | 2.69 | 8.0 | 7.77 | 2.97 | 0 | 13 | 13 | -0.57 | -0.02 | 0.18 |
| epiImp | 76 | 231 | 4.37 | 1.88 | 4.0 | 4.36 | 1.48 | 0 | 9 | 9 | 0.06 | -0.62 | 0.12 |
| epilie | 77 | 231 | 2.38 | 1.50 | 2.0 | 2.27 | 1.48 | 0 | 7 | 7 | 0.66 | 0.24 | 0.10 |
| ... | | | | | | | | | | | | | |
| traitanx | 85 | 231 | 39.01 | 9.52 | 38.0 | 38.36 | 8.90 | 22 | 71 | 49 | 0.67 | 0.47 | 0.63 |
| stateanx | 86 | 231 | 39.85 | 11.48 | 38.0 | 38.92 | 10.38 | 21 | 79 | 58 | 0.72 | -0.01 | 0.76 |



Multiple ways to graphically display data

1. box.plots (Core R)
2. Violin plots (`violinBy` in *psych*)
3. Scatter Plot Matrix (SPLOM) plots (`pairs.panels` in *psych*)

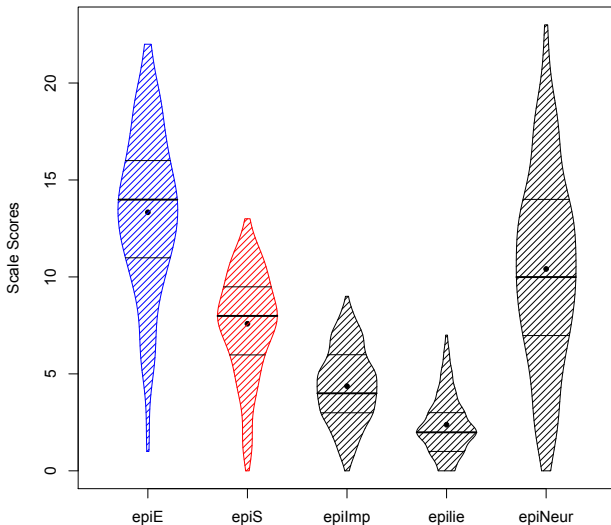
First, lets just make a smaller data.frame and then issue two different graphic commands.

R code

```
my.scales <- cleaned[74:86]
violinBy(my.scales[1:5])
pairs.panels(my.scales[c(1, 4, 5:10)], gap=0, pch=".")
```


Violin Plot `violinBy(my.scales[1:5])`

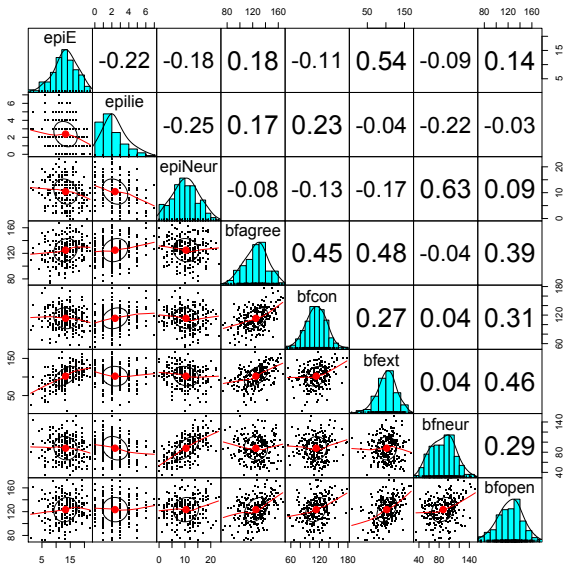
Violin Plot of the EPI data set





Scatter Plot of Matrices (SPLOM) of select variables

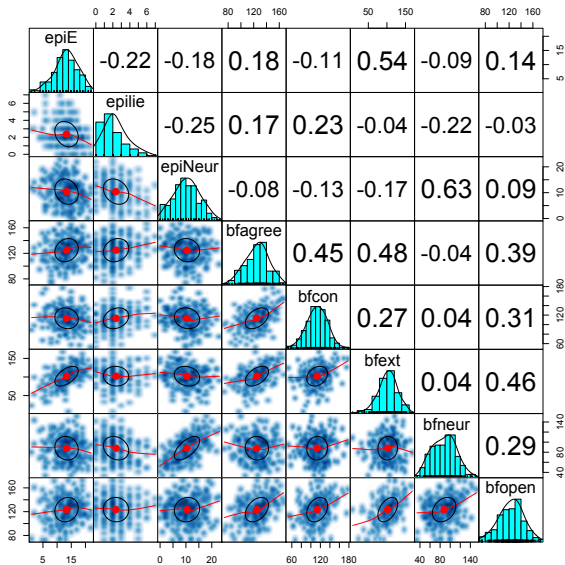
`pairs.panels(my.scales[c(1,4,5:10)],gap=0,pch=".")`





Scatter Plot of Matrices (SPLOM) of select variables

`pairs.panels(my.scales[c(1,4,5:10)],gap=0,pch=".",smoother=TRUE`





Show a table of correlations

R code

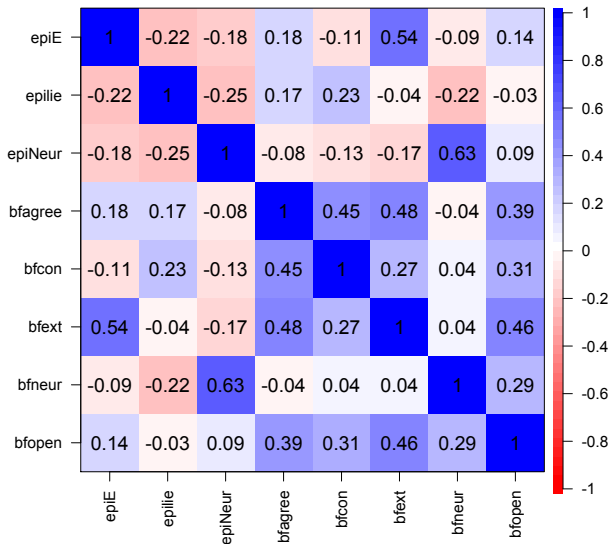
```
R <- lowerCor(my.scales[c(1,4,5:10)])
cor.plot(R, numbers=TRUE)
```

| | epiE | epili | epiNr | bfagr | bfcon | bfext | bfner | bfopn |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| epiE | 1.00 | | | | | | | |
| epilie | -0.22 | 1.00 | | | | | | |
| epiNeur | -0.18 | -0.25 | 1.00 | | | | | |
| bfagree | 0.18 | 0.17 | -0.08 | 1.00 | | | | |
| bfcon | -0.11 | 0.23 | -0.13 | 0.45 | 1.00 | | | |
| bfext | 0.54 | -0.04 | -0.17 | 0.48 | 0.27 | 1.00 | | |
| bfneur | -0.09 | -0.22 | 0.63 | -0.04 | 0.04 | 0.04 | 1.00 | |
| bfopen | 0.14 | -0.03 | 0.09 | 0.39 | 0.31 | 0.46 | 0.29 | 1.00 |

Automatically calls the `cor` and `round` functions with default parameters and then does a pretty print out using `lowerMat`. Invisibly returns the full (square) matrix of unrounded values.

A simple heat map using `cor.plot`

Correlation plot



Testing and displaying the “significance” of a set of correlations

1. Normal theory `corr.test`
 - raw probabilities as well as with a Holm adjusted for multiple correlations
2. Display these with `cor.plot`
3. Boot strapped confidence intervals based significance using `cor.ci`
 - Graphic displays correlations scaled by “significance”
 - Graphic displays of probability of correlation using `plot.cor.upperLowerCi`

Code for the next slides

R code

```
corr.test(my.scales[c(1,4,5:10)])
ci <- cor.ci(my.scales[c(1,4,5:10)])
cor.plot.upperLowerCi(ci). #temporarily unavailable -- patch coming in
```

Normal theory test of correlations using `corr.test`

R code

```
> corr.test(my.scales[c(1,4,5:10)])
```

Correlation matrix

| | epiE | epilie | epiNeur | bfagree | bfcon | bfext | bfneur | bfopen |
|---------|-------|--------|---------|---------|-------|-------|--------|--------|
| epiE | 1.00 | -0.22 | -0.18 | 0.18 | -0.11 | 0.54 | -0.09 | 0.14 |
| epilie | -0.22 | 1.00 | -0.25 | 0.17 | 0.23 | -0.04 | -0.22 | -0.03 |
| epiNeur | -0.18 | -0.25 | 1.00 | -0.08 | -0.13 | -0.17 | 0.63 | 0.09 |
| bfagree | 0.18 | 0.17 | -0.08 | 1.00 | 0.45 | 0.48 | -0.04 | 0.39 |
| bfcon | -0.11 | 0.23 | -0.13 | 0.45 | 1.00 | 0.27 | 0.04 | 0.31 |
| bfext | 0.54 | -0.04 | -0.17 | 0.48 | 0.27 | 1.00 | 0.04 | 0.46 |
| bfneur | -0.09 | -0.22 | 0.63 | -0.04 | 0.04 | 0.04 | 1.00 | 0.29 |
| bfopen | 0.14 | -0.03 | 0.09 | 0.39 | 0.31 | 0.46 | 0.29 | 1.00 |

Sample Size

[1] 231

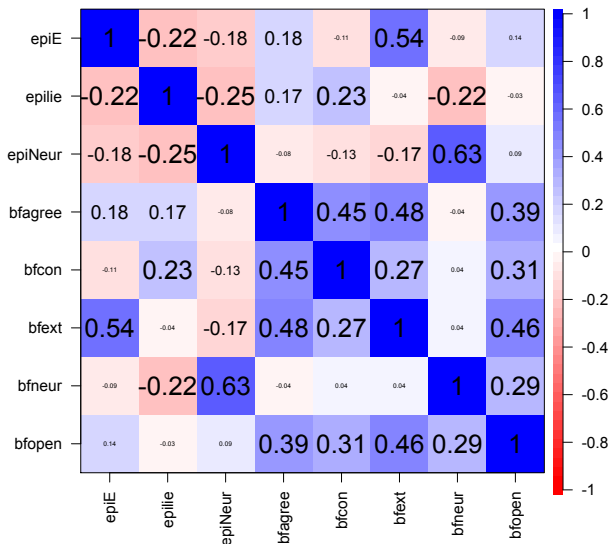
Probability values (Entries above the diagonal are adjusted for multiple tests.)

| | epiE | epilie | epiNeur | bfagree | bfcon | bfext | bfneur | bfopen |
|---------|------|--------|---------|---------|-------|-------|--------|--------|
| epiE | 0.00 | 0.01 | 0.11 | 0.11 | 0.75 | 0.00 | 1.00 | 0.4 |
| epilie | 0.00 | 0.00 | 0.00 | 0.12 | 0.01 | 1.00 | 0.01 | 1.0 |
| epiNeur | 0.01 | 0.00 | 0.00 | 1.00 | 0.43 | 0.12 | 0.00 | 1.0 |
| bfagree | 0.01 | 0.01 | 0.21 | 0.00 | 0.00 | 0.00 | 1.00 | 0.0 |
| bfcon | 0.08 | 0.00 | 0.04 | 0.00 | 0.00 | 0.00 | 1.00 | 0.0 |
| bfext | 0.00 | 0.50 | 0.01 | 0.00 | 0.00 | 0.00 | 1.00 | 0.0 |
| bfneur | 0.15 | 0.00 | 0.00 | 0.50 | 0.50 | 0.57 | 0.00 | 0.0 |
| bfopen | 0.04 | 0.70 | 0.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.0 |

To see confidence intervals of the correlations, print with the `short=FALSE` option

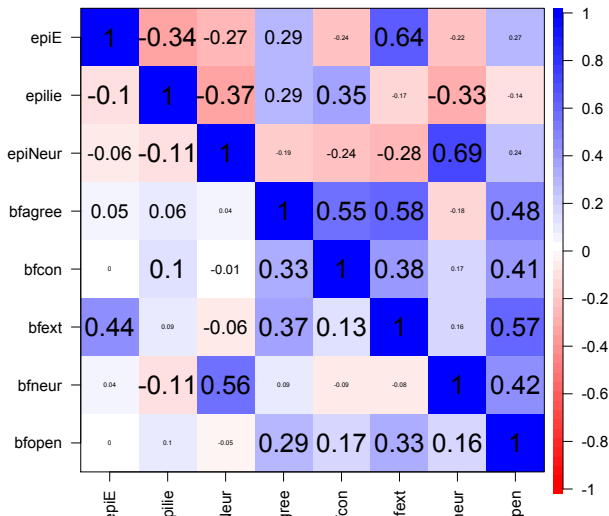
Heat map scaled by “significance” using `cor.ci`

Correlation plot



Heat map scaled by “significance” show upper and lower bounds–patched in 1.7.4

Upper and lower confidence intervals of correlations



Multiple types of reliability

1. Internal consistency estimates

- α, λ_6 , use the alpha function
- $\omega_{hierarchical}$ and ω_{total} use the omega function

2. IntraClass coefficients

- ICC

3. Rater agreement use kappa function

For the next examples we will use a built in data set

1. `bfi` consists of 25 personality items measuring 5 factors as well as some demographics.
2. The data were collected as part of the SAPA project and have 2,800 subjects.
3. For help on this data set, `?bfi`
4. To see all of the *psych* data sets: `data(package="psych")`

○○
○○○○○○○○
○○○○○●○○○
○○○○○○○○○○○ ○○○○○○○○
○○○○

○○○

First, we intentionally misspecify the data

R code

```
alpha(bfi[1:5]) #score the first five items
```

Some items (A1) were negatively correlated with the total scale and probably should be reversed.

To do this, run the function again with the 'check.keys=TRUE' option

Reliability analysis

Call: alpha(x = bfi[1:5])

```
raw_alpha std.alpha G6(smc) average_r S/N ase mean sd
0.43      0.46      0.53      0.15 0.85 0.016 4.2 0.74
```

```
lower alpha upper      95% confidence boundaries
0.4 0.43 0.46
```

Reliability if an item is dropped:

```
raw_alpha std.alpha G6(smc) average_r S/N alpha se
A1      0.72      0.73      0.67      0.398 2.64  0.0087
A2      0.28      0.30      0.39      0.097 0.43  0.0219
A3      0.18      0.21      0.31      0.061 0.26  0.0249
A4      0.25      0.31      0.44      0.099 0.44  0.0229
A5      0.21      0.24      0.36      0.072 0.31  0.0238
```

Item statistics

```
n raw.r std.r r.cor r.drop mean sd
A1 2784 0.066 0.024 -0.39 -0.31 2.4 1.4 <-- need to rekey this item
A2 2773 0.630 0.666 0.58 0.37 4.8 1.2
A3 2774 0.724 0.742 0.72 0.48 4.6 1.3
A4 2781 0.686 0.661 0.50 0.37 4.7 1.5
A5 2784 0.700 0.719 0.64 0.45 4.6 1.3
```

Non missing response frequency for each item

```
1 2 3 4 5 6 miss
```

Try it again. Turn on automatic reversals. Get the scores

R code

```
scores <- alpha(bfi[1:5], check.keys =TRUE)
```

```
alpha(bfi[1:5], check.keys =TRUE)
```

Reliability analysis

Call: alpha(x = bfi[1:5], check.keys = TRUE)

| raw_alpha | std.alpha | G6(smc) | average_r | S/N | ase | mean | sd |
|-----------|-----------|---------|-----------|-----|-------|------|-----|
| 0.7 | 0.71 | 0.68 | 0.33 | 2.5 | 0.009 | 4.7 | 0.9 |

lower alpha upper 95% confidence boundaries
0.69 0.7 0.72

Reliability if an item is dropped:

| | raw_alpha | std.alpha | G6(smc) | average_r | S/N | alpha | se |
|-----|-----------|-----------|---------|-----------|-----|--------|----|
| A1- | 0.72 | 0.73 | 0.67 | 0.40 | 2.6 | 0.0087 | |
| A2 | 0.62 | 0.63 | 0.58 | 0.29 | 1.7 | 0.0119 | |
| A3 | 0.60 | 0.61 | 0.56 | 0.28 | 1.6 | 0.0124 | |
| A4 | 0.69 | 0.69 | 0.65 | 0.36 | 2.3 | 0.0098 | |
| A5 | 0.64 | 0.66 | 0.61 | 0.32 | 1.9 | 0.0111 | |
| ... | | | | | | | |

Warning message:

In alpha(bfi[1:5], check.keys = TRUE) :

Some items were negatively correlated with total scale and were automatically reversed.
This is indicated by a negative sign for the variable name.

R functions will return objects without necessarily telling you

1. The basic logic of R is that you can do lots of calculations, but you might not want all the output.
2. The output is there, to be processed by other functions if you want, but you probably don't want to see all of it unless you ask.,
3. Thus, `alpha` returns the scores based upon the scales you asked for, but doesn't show them, because they are so many,
4. The `str` command tells you the structure of an object. The `names` will just list the names of the objects.

names and str of alpha output

R code

```
names(scores)
str(scores)
```

```
names(scores)
[1] "total"          "alpha.drop"      "item.stats"      "response.freq"  "keys"
     "scores"        "nvar"            "boot.ci"
[9] "boot"          "Unidim"          "Fit"              "call"           "title"

    $ total          : 'data.frame':      1 obs. of  8 variables:
..$ raw_alpha: num 0.703
..$ std.alpha: num 0.713
..$ G6(smc) : num 0.683
..$ average_r: num 0.332
..$ S/N : num 2.48
..$ ase : num 0.00895
..$ mean : num 4.65
..$ sd : num 0.898
$ alpha.drop : 'data.frame':      5 obs. of  6 variables:
..$ raw_alpha: num [1:5] 0.719 0.617 0.6 0.686 0.643
..$ std.alpha: num [1:5] 0.726 0.626 0.613 0.694 0.656
..$ G6(smc) : num [1:5] 0.673 0.579 0.558 0.65 0.605
..$ average_r: num [1:5] 0.398 0.295 0.284 0.361 0.322
..$ S/N : num [1:5] 2.64 1.67 1.58 2.26 1.9
..$ alpha se : num [1:5] 0.00873 0.0119 0.01244 0.00983 0.01115
$ item.stats : 'data.frame':      5 obs. of  7 variables:
..$ n : num [1:5] 2784 2773 2774 2781 2784
..$ raw.r : num [1:5] 0.581 0.728 0.76 0.654 0.687
..$ std.r : num [1:5] 0.566 0.748 0.767 0.631 0.699
..$ r.cor : num [1:5] 0.376 0.667 0.709 0.471 0.596
..$ r.drop: num [1:5] 0.308 0.564 0.587 0.394 0.489
..$ mean : num [1:5] 4.59 4.8 4.6 4.7 4.56
..$ sd : num [1:5] 1.41 1.17 1.3 1.48 1.26
```

One of the objects of alpha is the scores object

R code

```
describe(scores$scores)
```

But, since there scores for all subjects, but just one score, this is not very interesting.

```
describe(scores$scores)
  vars      n mean  sd median trimmed  mad min max range  skew kurtosis
x1    1 2800 4.65 0.9   4.8   4.73 0.89   1  6   5 -0.76
>
```

Note that alpha has the option of doing cumulative scores (adding up items, or scoring in the unit of the items (the default).

R code

```
scores <- alpha(bfi[1:5], check.keys=TRUE, cumulative=TRUE)
#set the cumulative option to be true
describe(scores$scores)
```

```
describe(scores$scores)
  vars      n mean  sd median trimmed  mad min max range  skew kurtosis
x1    1 2800 23.08 4.54   24  23.43 4.45   5 30  25 -0.73
```


Perhaps a more useful case: scoring multiple scales using scoreItems

First, define the scoring keys, and then call scoreItems Use the
msq data set

R code

```
keys <- make.keys(msq[1:75], list(
+ EA = c("active", "energetic", "vigorous", "wakeful", "wide.awake", "full.of.pep",
+       "lively", "-sleepy", "-tired", "-drowsy"),
+ TA = c("intense", "jittery", "fearful", "tense", "clutched.up", "-quiet", "-still",
+       "-placid", "-calm", "-at.rest"),
+ PA = c("active", "excited", "strong", "inspired", "determined", "attentive",
+       "interested", "enthusiastic", "proud", "alert"),
+ Naf = c("jittery", "nervous", "scared", "afraid", "guilty", "ashamed", "distressed",
+       "upset", "hostile", "irritable" ))
+ )
> msq.scores <- scoreItems(keys, msq[1:75])
> msq.scores
```

```
all: scoreItems(keys = keys, items = msq[1:75])
```

(Unstandardized) Alpha:

| | EA | TA | PA | NAf |
|-------|------|------|------|------|
| alpha | 0.93 | 0.75 | 0.92 | 0.83 |

Standard errors of unstandardized Alpha:

| | EA | TA | PA | NAf |
|-----|-------|--------|--------|--------|
| ASE | 0.004 | 0.0082 | 0.0044 | 0.0064 |

Average item correlation:

| | EA | TA | PA | NAf |
|-----------|------|------|------|------|
| average r | 0.58 | 0.23 | 0.52 | 0.33 |

Score multiple scales (continued)

Standard errors of unstandardized Alpha:

| | EA | TA | PA | NAf |
|-----|-------|--------|--------|--------|
| ASE | 0.004 | 0.0082 | 0.0044 | 0.0064 |

Average item correlation:

| | EA | TA | PA | NAf |
|-----------|------|------|------|------|
| average.r | 0.58 | 0.23 | 0.52 | 0.33 |

Guttman 6* reliability:

| | EA | TA | PA | NAf |
|----------|------|------|------|------|
| Lambda.6 | 0.96 | 0.82 | 0.93 | 0.88 |

Signal/Noise based upon av.r :

| | EA | TA | PA | NAf |
|--------------|----|----|----|-----|
| Signal/Noise | 14 | 3 | 11 | 4.9 |

Scale intercorrelations corrected for attenuation

raw correlations below the diagonal, alpha on the diagonal
corrected correlations above the diagonal:

| | EA | TA | PA | NAf |
|-----|--------|------|-------|--------|
| EA | 0.932 | 0.29 | 0.870 | -0.069 |
| TA | 0.238 | 0.75 | 0.226 | 0.710 |
| PA | 0.804 | 0.19 | 0.915 | 0.044 |
| NAf | -0.061 | 0.56 | 0.039 | 0.831 |

More detailed item statistics

R code

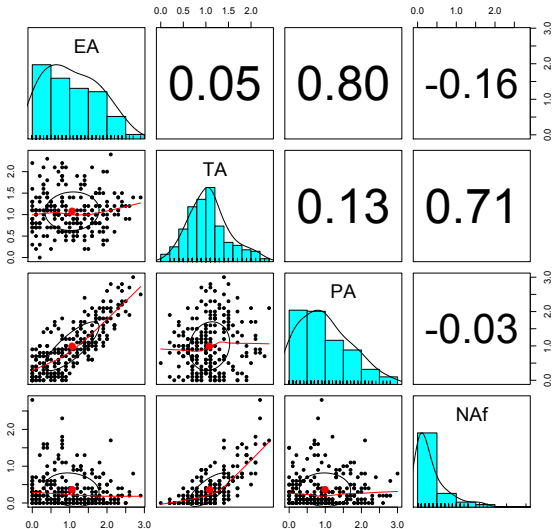
```
print(msq.scores, short=FALSE)
```

| Item by scale correlations: corrected for item overlap and scale reliability | | | | | Non missing response frequency for each item 0 1 2 3 miss | | | | | |
|---|-------|-------|-------|-------|--|------|------|------|------|------|
| | EA | TA | PA | NAF | | | | | | |
| delighted | 0.56 | 0.04 | 0.67 | -0.17 | delighted | 0.50 | 0.27 | 0.19 | 0.04 | 0.00 |
| sociable | 0.57 | 0.06 | 0.64 | -0.15 | sociable | 0.24 | 0.30 | 0.35 | 0.11 | 0.00 |
| jittery | 0.15 | 0.52 | 0.23 | 0.46 | jittery | 0.60 | 0.28 | 0.09 | 0.03 | 0.00 |
| hostile | -0.21 | 0.37 | -0.13 | 0.58 | hostile | 0.77 | 0.17 | 0.05 | 0.01 | 0.00 |
| sluggish | -0.65 | -0.02 | -0.40 | 0.21 | sluggish | 0.24 | 0.44 | 0.19 | 0.13 | 0.00 |
| depressed | -0.30 | 0.44 | -0.26 | 0.67 | depressed | 0.61 | 0.26 | 0.08 | 0.05 | 0.00 |
| satisfied | 0.54 | -0.23 | 0.62 | -0.35 | satisfied | 0.22 | 0.29 | 0.38 | 0.12 | 0.00 |
| relaxed | 0.35 | -0.56 | 0.40 | -0.51 | relaxed | 0.13 | 0.20 | 0.43 | 0.24 | 0.00 |
| warmhearted | 0.47 | -0.09 | 0.66 | -0.19 | warmhearted | 0.17 | 0.23 | 0.37 | 0.22 | 0.00 |
| blue | -0.23 | 0.40 | -0.19 | 0.64 | blue | 0.60 | 0.30 | 0.08 | 0.02 | 0.00 |
| intense | 0.25 | 0.42 | 0.44 | 0.47 | intense | 0.54 | 0.28 | 0.15 | 0.03 | 0.00 |
| strong | 0.55 | 0.00 | 0.69 | -0.03 | strong | 0.31 | 0.26 | 0.32 | 0.11 | 0.00 |
| scared | -0.05 | 0.61 | 0.08 | 0.75 | scared | 0.80 | 0.15 | 0.04 | 0.01 | 0.00 |
| enthusiastic | 0.69 | 0.16 | 0.83 | -0.07 | enthusiastic | 0.43 | 0.29 | 0.19 | 0.08 | 0.00 |
| proud | 0.57 | 0.05 | 0.73 | -0.11 | proud | 0.41 | 0.24 | 0.26 | 0.10 | 0.00 |
| sad | -0.22 | 0.46 | -0.18 | 0.73 | sad | 0.66 | 0.22 | 0.09 | 0.03 | 0.00 |
| active | 0.71 | 0.18 | 0.83 | -0.05 | active | 0.38 | 0.32 | 0.22 | 0.08 | 0.00 |
| full.of.pep | 0.80 | 0.05 | 0.80 | -0.15 | full.of.pep | 0.54 | 0.21 | 0.20 | 0.05 | 0.00 |
| unhappy | -0.30 | 0.44 | -0.26 | 0.70 | unhappy | 0.65 | 0.24 | 0.07 | 0.04 | 0.00 |
| lively | 0.78 | 0.04 | 0.81 | -0.13 | lively | 0.44 | 0.28 | 0.22 | 0.06 | 0.00 |



Show the SPLOM of the msq scales using `pairs.panels`

Four msq scales



But what if we have overlapping scales?

1. Sometimes we are interested in how higher order scales relate to lower order scales.
2. The problem is, the items overlap.
3. Some people solve this problem by dropping the overlapping items. But this changes the meaning of the scales.
4. A fairly straightforward procedure is estimate the overlapping variances with the best estimate of shared (common) variance, similar to what is done when finding coefficient α .
5. Need to do this on the correlation matrix of the items, not the raw data.
6. See ?scoreOverlap

Correcting for item overlap using scoreOverlap

R code

```

small.msq <- msq[ c("active", "energetic", "vigorous", "wakeful", "wide.awake", "full.of.pep", "lively", "sleepy", "tired", "drowsy", "intense", "jittery", "tense", "clutched.up", "quiet", "still", "placid", "calm", "at.rest")
small.R <- cor(small.msq,use="pairwise")
keys.list <- list(
EA = c("active", "energetic", "vigorous", "wakeful", "wide.awake", "full.of.pep", "lively", "-sleepy", "-tired", "-drowsy"),
TA =c("intense", "jittery", "fearful", "tense", "clutched.up", "quiet", "-placid", "-calm", "-at.rest") ,

high.EA = c("active", "energetic", "vigorous", "wakeful", "wide.awake", "full.of.pep", "lively"),
low.EA =c("sleepy", "tired", "drowsy"),
lowTA= c("quiet", "still", "placid", "calm", "at.rest"),
highTA = c("intense", "jittery", "fearful", "tense", "clutched.up")
)

keys <- make.keys(small.R,keys.list)

adjusted.scales <- scoreOverlap(keys.list,small.R)

```

Correcting for item overlap using scoreOverlap. (continued)

Call: scoreOverlap(keys = keys.list, r = small.R)

(Standardized) Alpha:

| EA | TA | high.EA | low.EA | lowTA | highTA |
|------|------|---------|--------|-------|--------|
| 0.93 | 0.75 | 0.94 | 0.93 | 0.73 | 0.76 |

(Standardized) G6*:

| EA | TA | high.EA | low.EA | lowTA | highTA |
|------|------|---------|--------|-------|--------|
| 0.88 | 0.68 | 0.94 | 0.90 | 0.73 | 0.75 |

Average item correlation:

| EA | TA | high.EA | low.EA | lowTA | highTA |
|------|------|---------|--------|-------|--------|
| 0.59 | 0.23 | 0.68 | 0.81 | 0.35 | 0.38 |

Number of items:

| EA | TA | high.EA | low.EA | lowTA | highTA |
|----|----|---------|--------|-------|--------|
| 10 | 10 | 7 | 3 | 5 | 5 |

Signal to Noise ratio based upon average r and n

| EA | TA | high.EA | low.EA | lowTA | highTA |
|------|-----|---------|--------|-------|--------|
| 14.1 | 3.0 | 14.8 | 12.9 | 2.7 | 3.1 |

Scale intercorrelations corrected for item overlap and attenuation

adjusted for overlap correlations below the diagonal, alpha on the diagonal
corrected correlations above the diagonal:

| | EA | TA | high.EA | low.EA | lowTA | highTA |
|---------|-------|-------|---------|--------|-------|--------|
| EA | 0.93 | 0.27 | 0.965 | -0.803 | -0.18 | 0.253 |
| TA | 0.23 | 0.75 | 0.282 | -0.167 | -0.81 | 0.821 |
| high.EA | 0.90 | 0.24 | 0.937 | -0.620 | -0.12 | 0.324 |
| low.EA | -0.75 | -0.14 | -0.579 | 0.928 | 0.25 | -0.023 |
| lowTA | -0.15 | -0.60 | -0.098 | 0.204 | 0.73 | -0.335 |
| highTA | 0.21 | 0.62 | 0.273 | -0.019 | -0.25 | 0.757 |

Compare scoreOverlap with non-adjusted

R code

```
adjusted.scales <- scoreOverlap(keys.list, small.R)
raw <- scoreItems(keys.list, small.R)
```

Scale intercorrelations corrected for item overlap and attenuation
adjusted for overlap correlations below the diagonal, alpha on the diagonal
corrected correlations above the diagonal:

| | EA | TA | high.EA | low.EA | lowTA | highTA |
|---------|-------|-------|---------|--------|-------|--------|
| EA | 0.93 | 0.27 | 0.965 | -0.803 | -0.18 | 0.253 |
| TA | 0.23 | 0.75 | 0.282 | -0.167 | -0.81 | 0.821 |
| high.EA | 0.90 | 0.24 | 0.937 | -0.620 | -0.12 | 0.324 |
| low.EA | -0.75 | -0.14 | -0.579 | 0.928 | 0.25 | -0.023 |
| lowTA | -0.15 | -0.60 | -0.098 | 0.204 | 0.73 | -0.335 |
| highTA | 0.21 | 0.62 | 0.273 | -0.019 | -0.25 | 0.757 |

Scale intercorrelations corrected for attenuation
raw correlations below the diagonal, alpha on the diagonal
corrected correlations above the diagonal:

| | EA | TA | high.EA | low.EA | lowTA | highTA |
|---------|-------|-------|---------|--------|-------|--------|
| EA | 0.93 | 0.27 | 1.024 | -0.848 | -0.18 | 0.253 |
| TA | 0.23 | 0.75 | 0.282 | -0.167 | -1.06 | 1.056 |
| high.EA | 0.96 | 0.24 | 0.937 | -0.620 | -0.12 | 0.324 |
| low.EA | -0.79 | -0.14 | -0.579 | 0.928 | 0.25 | -0.023 |
| lowTA | -0.15 | -0.78 | -0.098 | 0.204 | 0.73 | -0.335 |
| highTA | 0.21 | 0.80 | 0.273 | -0.019 | -0.25 | 0.757 |

α , $\omega_{hierarchical}$ and β as alternative measures of internal consistency

1. α as the mean split half reliability
 - alpha to find α
 - `splitHalf` to find all (if $n \leq 16$) or 10,000 random possible split half reliabilities ($n > 16$)
2. $\omega_{hierarchical}$ and ω_{total} as factor based reliabilities
 - $\omega_{hierarchical}$ estimates general factor saturation
 - Found using `omega` and `omegaSem`
3. β as worst split half reliability as an alternative estimate of the general factor saturation.
 - Found using a hierarchical clustering algorithm (`iclust`).
 - `iclust` is also useful for scale construction.

α from alpha and all split halves found using splitHalf

Find α and all split half reliabilities of 5 Agreeableness items and 5 Conscientiousness items from the `bfi` data set included in *psych*.

R code

```
alpha(bfi[1:10]) #find alpha, let it automatically reverse items
splitHalf(bfi[1:10], keys=c(1,9,10)) #reverse 3 items
```

Reliability analysis

```
Call: alpha(x = bfi[1:10])
```

```
raw_alpha std.alpha G6(smc) average_r S/N ase mean sd
0.73      0.74      0.76      0.22 2.8 0.01 4.5 0.73
```

```
lower alpha upper      95% confidence boundaries
```

```
0.71 0.73 0.75
```

Split half reliabilities

```
Call: splitHalf(r = bfi[1:10], keys = c(1, 9, 10))
```

```
Maximum split half reliability (lambda 4) = 0.81
```

```
Guttman lambda 6 = 0.76
```

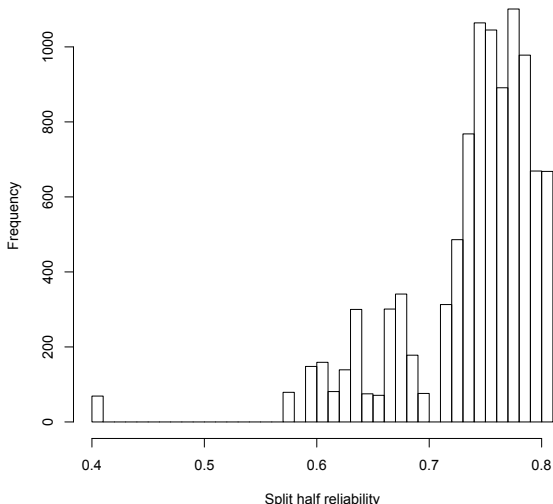
```
Average split half reliability = 0.73
```

```
Guttman lambda 3 (alpha) = 0.74
```

```
Minimum split half reliability (beta) = 0.41
```

All possible split halves of 5 agreeableness and 5 conscientiousness items. Note the one worst one! This is not one construct.

All split half reliabilities of bfi[1:10]



○○
○○○○○○○○
○○○○○○○○○○
○○○○○○○●○○○ ○○○○○○○○
○○○○

○○○

Using the ω function

R code

`omega(ability, 4)`

Omega

Call: `omega(m = ability, nfactores = 4)`

Alpha: 0.83

G.6: 0.84

Omega Hierarchical: 0.65

Omega H asymptotic: 0.76

Omega Total 0.86

Schmid Leiman Factor loadings greater than 0.2

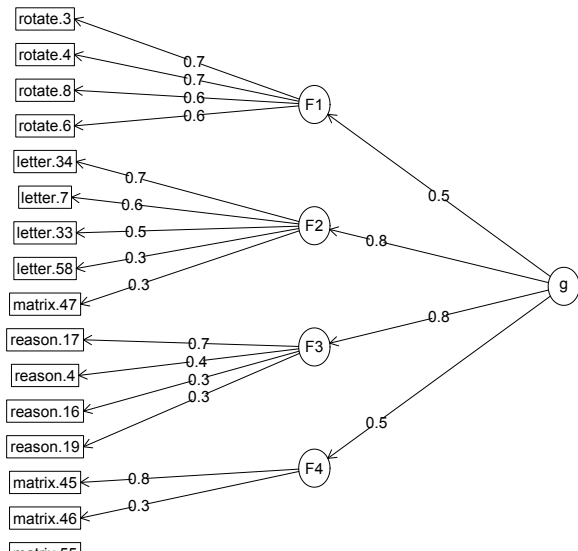
| | g | F1* | F2* | F3* | F4* | h2 | u2 | p2 |
|-----------|------|------|------|------|-----|------|------|------|
| reason.4 | 0.50 | | | 0.27 | | 0.34 | 0.66 | 0.73 |
| reason.16 | 0.42 | | | 0.21 | | 0.23 | 0.77 | 0.76 |
| reason.17 | 0.55 | | | 0.47 | | 0.52 | 0.48 | 0.57 |
| reason.19 | 0.44 | | | 0.21 | | 0.25 | 0.75 | 0.77 |
| letter.7 | 0.52 | | 0.35 | | | 0.39 | 0.61 | 0.69 |
| letter.33 | 0.46 | | 0.30 | | | 0.31 | 0.69 | 0.70 |
| letter.34 | 0.54 | | 0.38 | | | 0.43 | 0.57 | 0.67 |
| letter.58 | 0.47 | | 0.20 | | | 0.28 | 0.72 | 0.78 |
| matrix.45 | 0.40 | | | 0.66 | | 0.59 | 0.41 | 0.27 |
| matrix.46 | 0.40 | | | 0.26 | | 0.24 | 0.76 | 0.65 |
| matrix.47 | 0.42 | | | | | 0.23 | 0.77 | 0.79 |
| matrix.55 | 0.28 | | | | | 0.12 | 0.88 | 0.65 |
| rotate.3 | 0.36 | 0.61 | | | | 0.50 | 0.50 | 0.26 |
| rotate.4 | 0.41 | 0.61 | | | | 0.54 | 0.46 | 0.31 |
| rotate.6 | 0.40 | 0.49 | | | | 0.41 | 0.59 | 0.39 |
| rotate.8 | 0.32 | 0.53 | | | | 0.40 | 0.60 | 0.26 |

With eigenvalues of:

| g | F1* | F2* | F3* | F4* |
|------|------|------|------|------|
| 3.04 | 1.32 | 0.46 | 0.42 | 0.55 |

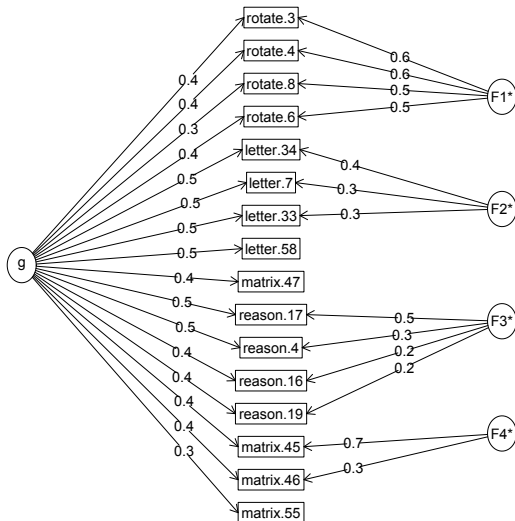
16 ability items from the International Cognitive Ability Resource

general ability and 4 subfactors of ICAR data



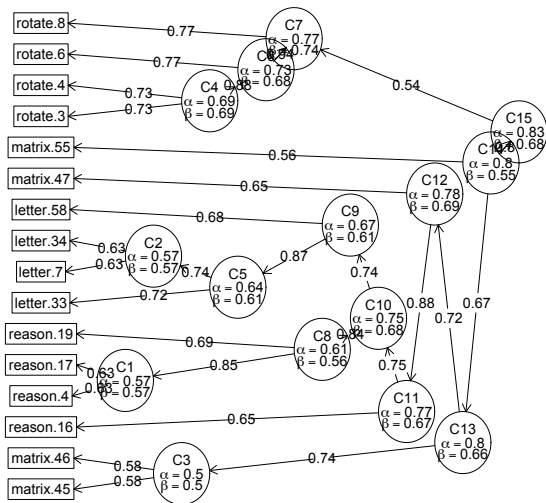
Schmid Leiman transformation of 16 ability items from ICAR

general ability and 4 subfactors of ICAR data



Hierarchical clustering of 16 ICAR ability items: iclust(ability)

Hierarchical clustering of 16 ability items using iclust



Exploratory Factor Analysis

- How many factors: an unsolved problem
 - Parallel analysis, MAPS, VSS, BIC, RMSEA, etc. available in `nfactors` and `fa.parallel`
- Factor extraction algorithms available in the `fa` function
 - maximum likelihood, minimum residual, principal factor, ...
- Factor rotation procedures are done using *GPArotation* package
 - orthogonal: `varimax`, `quartimax`, `bifactor`, ...
 - oblique: `oblimin`, `geomin`, `biquartimin`, ...
- Displaying the solutions using `fa.plot`

Note, that EFA is not the same as Principal Components Analysis and the two should not be confused.

- PCA done using `principal`

The number of factors problem is easy and hard

No best rule, one worst rule

"Solving the number of factors problem is easy, I do it everyday before breakfast. But knowing the right solution is harder." (Henry Kaiser)

1. Parallel analysis (Extract factors until the eigen values are less than those of a random matrix).
 - Although a good rule for 100-500 subjects, this will not do as well with many (>1000) subjects.
2. Velicer's Minimum Average Partial (MAP) is pretty good
3. For items, the Very Simple Structure (VSS) criterion is pretty good.
4. Multiple statistical tests, many have problems with sample size.
 - If you want few factors, run few subjects
 - If you want many factors, run many subjects
5. One worst rule is the eigen value of 1.0 rule.



What about parallel analysis? Pearson R or polychoric ρ ?

Of the raw (Pearson) correlations compared to the polychoric correlations

R code

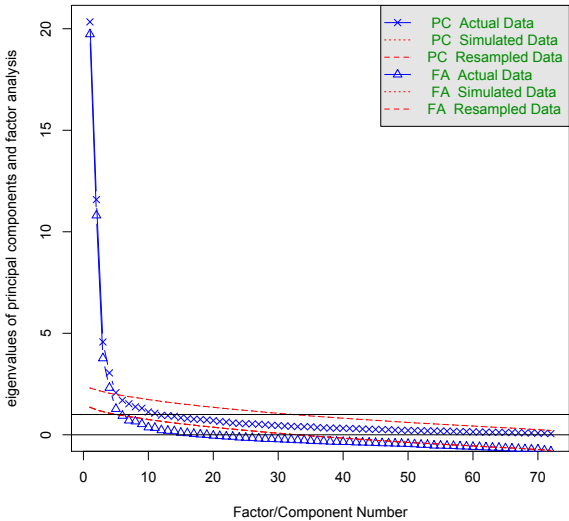
```
> fa.parallel(cleaned[2:73])
Parallel analysis suggests that the number of factors = 5
and the number of components = 5

#use polychoric correlations
fa.parallel(cleaned[2:73], cor="poly")

> fa.parallel(cleaned[2:73], cor="poly")
some warnings are issued
The items do not have an equal number of response
alternatives, global set to FALSE
Parallel analysis suggests that the number of factors = 4
and the number of components = 4
Warning message:
In cor.smooth(mat) : Matrix was not positive definite,
smoothing was done
>
```

Parallel analysis with Pearson correlations

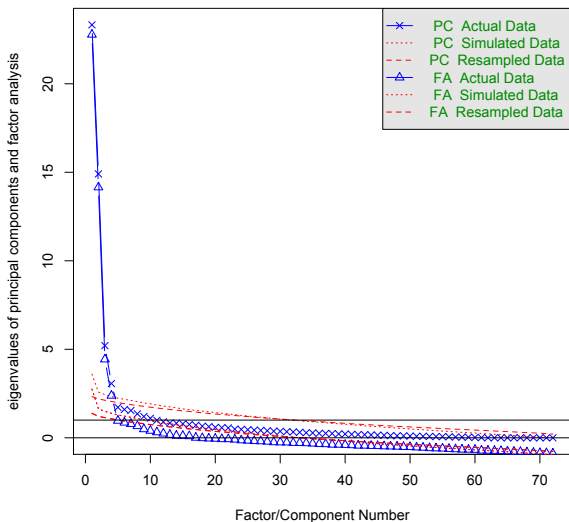
Parallel Analysis Scree Plots





Parallel analysis with polychoric correlations (takes somewhat longer)

Parallel Analysis Scree Plots



How many factors: what does `nfactors` tell us?

```
> nfactors(cleaned[2:73])
```

Number of factors

Call: `vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,`

`n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)`

VSS complexity 1 achieves a maximum of 0.74 with 2 factors

VSS complexity 2 achieves a maximum of 0.91 with 3 factors

The Velicer MAP achieves a minimum of 0.01 with 9 factors

Empirical BIC achieves a minimum of -10081.25 with 6 factors

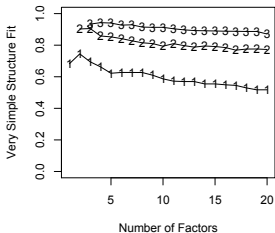
Sample Size adjusted BIC achieves a minimum of -1843.62 with 11 factors

Statistics by number of factors

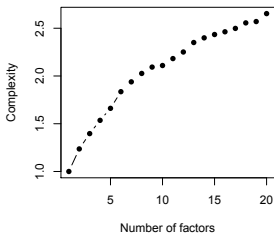
| | vss1 | vss2 | map | dof | chisq | prob | sqresid | fit | RMSEA | BIC | SABIC | complex | eChisq | SRMR |
|-----|------|------|--------|------|-------|----------|---------|------|-------|-------|-------|---------|--------|-------|
| 1 | 0.68 | 0.00 | 0.0552 | 2484 | 8907 | 0.0e+00 | 192.3 | 0.68 | 0.115 | -4612 | 3261 | 1.0 | 35253 | 0.173 |
| 2 | 0.74 | 0.90 | 0.0179 | 2413 | 5827 | 2.9e-282 | 58.9 | 0.90 | 0.087 | -7305 | 343 | 1.2 | 7619 | 0.080 |
| 3 | 0.69 | 0.91 | 0.0145 | 2343 | 4928 | 8.6e-186 | 40.3 | 0.93 | 0.077 | -7823 | -397 | 1.4 | 4442 | 0.061 |
| 4 | 0.66 | 0.86 | 0.0110 | 2274 | 4095 | 1.7e-107 | 29.0 | 0.95 | 0.067 | -8281 | -1074 | 1.5 | 2532 | 0.046 |
| 5 | 0.62 | 0.85 | 0.0101 | 2206 | 3587 | 1.9e-69 | 24.9 | 0.96 | 0.061 | -8419 | -1427 | 1.7 | 1933 | 0.040 |
| 6 | 0.63 | 0.84 | 0.0094 | 2139 | 3274 | 4.1e-51 | 22.0 | 0.96 | 0.057 | -8367 | -1588 | 1.8 | 1560 | 0.036 |
| 7 | 0.63 | 0.83 | 0.0094 | 2073 | 3047 | 2.1e-40 | 19.7 | 0.97 | 0.055 | -8235 | -1665 | 1.9 | 1314 | 0.033 |
| 8 | 0.63 | 0.82 | 0.0091 | 2008 | 2810 | 7.0e-30 | 17.9 | 0.97 | 0.052 | -8118 | -1754 | 2.0 | 1100 | 0.031 |
| ... | | | | | | | | | | | | | | |
| 18 | 0.53 | 0.78 | 0.0105 | 1413 | 1467 | 1.5e-01 | 9.2 | 0.98 | 0.032 | -6223 | -1745 | 2.6 | 324 | 0.017 |
| 19 | 0.52 | 0.78 | 0.0108 | 1359 | 1371 | 4.0e-01 | 8.7 | 0.99 | 0.030 | -6025 | -1718 | 2.6 | 291 | 0.016 |
| 20 | 0.52 | 0.77 | 0.0111 | 1306 | 1284 | 6.6e-01 | 8.2 | 0.99 | 0.028 | -5824 | -1685 | 2.7 | 258 | 0.015 |

The number of factors from `nfact`

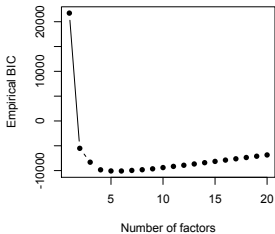
Very Simple Structure



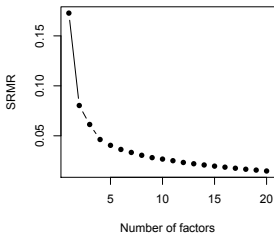
Complexity



Empirical BIC

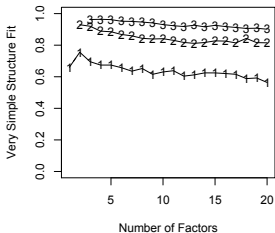


Root Mean Residual

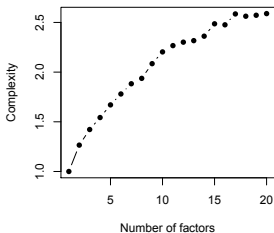


The number of factors from `nfactors`

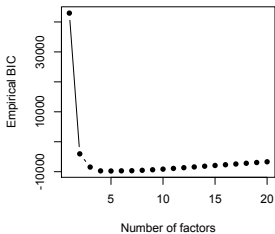
Very Simple Structure



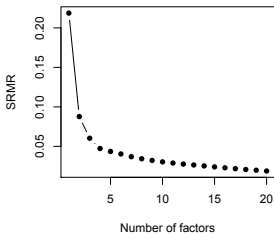
Complexity



Empirical BIC



Root Mean Residual



EFA of the Motivational State Questionnaire

R code

```
f2 <- fa(msq[1:72], 2)
summary(f2)
```

```
ummary(f2)
```

```
Factor analysis with Call: fa(r = msq[1:72], nfactors = 2)
```

```
Test of the hypothesis that 2 factors are sufficient.
```

```
The degrees of freedom for the model is 2413 and the objective fun
```

```
The number of observations was 3896 with Chi Square = 67730.13
```

```
The root mean square of the residuals (RMSA) is 0.09
```

```
The df corrected root mean square of the residuals is 0.09
```

```
Tucker Lewis Index of factoring reliability = 0.637
```

```
RMSEA index = 0.083 and the 90 % confidence intervals are 0.083
```

```
BIC = 47780.16
```

```
With factor correlations of
```

```
      MR1  MR2
MR1  1.00 -0.13
MR2 -0.13  1.00
```

Show the factors, sorted by factor loadings

```
> print(f2, sort=TRUE)
Factor Analysis using method = minres
Call: fa(r = msq[1:72], nfactors = 2, cor = "poly")
Standardized loadings (pattern matrix) based upon correlation matrix
```

| | item | MR1 | MR2 | h2 | u2 | com | |
|--|-------------|-----|-------|-------|-------|------|-----|
| | lively | 20 | 0.89 | -0.05 | 0.811 | 0.19 | 1.0 |
| | energetic | 55 | 0.89 | 0.05 | 0.789 | 0.21 | 1.0 |
| | full.of.pep | 18 | 0.89 | -0.05 | 0.800 | 0.20 | 1.0 |
| | ... | | | | | | |
| | sluggish | 5 | -0.52 | 0.22 | 0.348 | 0.65 | 1.4 |
| | sleepy | 59 | -0.48 | 0.15 | 0.274 | 0.73 | 1.2 |
| | tired | 28 | -0.45 | 0.23 | 0.285 | 0.71 | 1.5 |
| | drowsy | 51 | -0.40 | 0.13 | 0.189 | 0.81 | 1.2 |
| | ... | | | | | | |
| | tense | 69 | 0.14 | 0.85 | 0.714 | 0.29 | 1.1 |
| | frustrated | 65 | -0.10 | 0.83 | 0.718 | 0.28 | 1.0 |
| | ashamed | 70 | 0.12 | 0.83 | 0.676 | 0.32 | 1.0 |
| | upset | 48 | -0.13 | 0.82 | 0.714 | 0.29 | 1.1 |
| | ... | | | | | | |
| | relaxed | 8 | 0.44 | -0.52 | 0.519 | 0.48 | 1.9 |
| | calm | 50 | 0.26 | -0.50 | 0.354 | 0.65 | 1.5 |
| | at.rest | 26 | 0.38 | -0.43 | 0.378 | 0.62 | 2.0 |
| | ... | | | | | | |

| | MR1 | MR2 |
|-----------------------|-------|-------|
| SS loadings | 21.07 | 17.15 |
| Proportion Var | 0.29 | 0.24 |
| Cumulative Var | 0.29 | 0.53 |
| Proportion Explained | 0.55 | 0.45 |
| Cumulative Proportion | 0.55 | 1.00 |

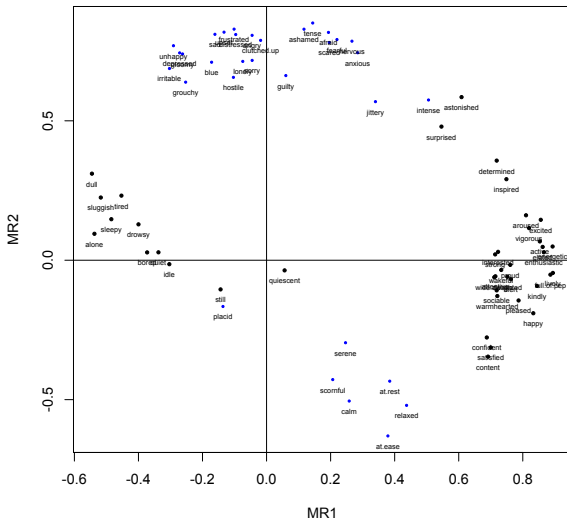
With factor correlations of

MR1 MR2



fa.plot(f2, labels=colnames(msq[1:72]), cex=.5, title="2 dimensions of the Motivational State Questionnaire")

2 dimensions of the Motivational State Questionnaire

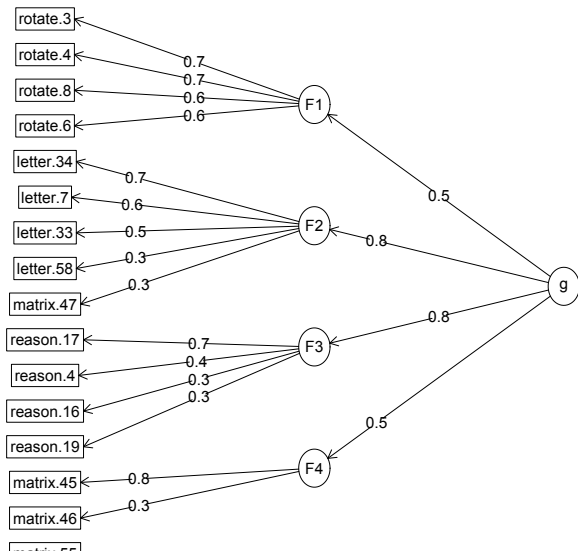


Convert to and sort polar coordinates `round(polar(f2), 2)`

| | Var | theta21 | vecl21 |
|---------------------|-----|---------|--------|
| strong | 12 | 1.68 | 0.51 |
| enthusiastic | 14 | 1.89 | 0.75 |
| ... | | | |
| anxious | 71 | 69.09 | 0.63 |
| nervous | 45 | 71.26 | 0.69 |
| ... | | | |
| angry | 44 | 93.20 | 0.65 |
| sorry | 58 | 93.59 | 0.52 |
| ... | | | |
| sad | 16 | 101.25 | 0.68 |
| blue | 10 | 103.59 | 0.53 |
| ... | | | |
| drowsy | 51 | 162.17 | 0.18 |
| sleepy | 59 | 163.11 | 0.26 |
| ... | | | |
| relaxed | 8 | 310.03 | 0.46 |
| at.rest | 26 | 311.56 | 0.34 |
| .. | | | |
| happy | 61 | 347.14 | 0.73 |
| pleased | 60 | 349.61 | 0.64 |
| ... | | | |
| alert | 52 | 354.89 | 0.59 |
| wide.awake | 64 | 355.09 | 0.51 |

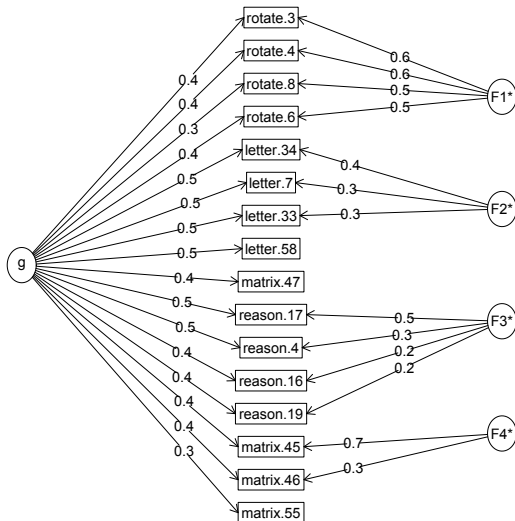
16 ability items from the International Cognitive Ability Resource

general ability and 4 subfactors of ICAR data



Schmid Leiman transformation of 16 ability items from ICAR

general ability and 4 subfactors of ICAR data



More types of reliability

1. α is a hodgepodge ratio of general factor and group factor reliability
2. ω_h (omega hierarchical) is an estimate of the general factor variance of a test
3. ω_t (omega total) is an estimate of the total reliable variance of a test
4. When do we use these?
 - When estimating how much of a test measures one thing.
omega_h
 - When estimating what is the total reliable variance in a test (when adjusting for test reliability in an SEM context)

ω_h and ω_t reliabilities

R code

```
om <- omega(ability, nfact=4)
```

```
om
Omega
Call: omega(m = ability, nfact = 4)
Alpha:          0.83
G.6:           0.84
Omega Hierarchical: 0.65
Omega H asymptotic: 0.76
Omega Total     0.86

Schmid Leiman Factor loadings greater than 0.2
```

| | g | F1* | F2* | F3* | F4* | h2 | u2 | p2 |
|-----------|------|------|------|------|------|------|------|------|
| reason.4 | 0.50 | | | 0.27 | | 0.34 | 0.66 | 0.73 |
| reason.16 | 0.42 | | | 0.21 | | 0.23 | 0.77 | 0.76 |
| reason.17 | 0.55 | | | 0.47 | | 0.52 | 0.48 | 0.57 |
| reason.19 | 0.44 | | | 0.21 | | 0.25 | 0.75 | 0.77 |
| letter.7 | 0.52 | | 0.35 | | | 0.39 | 0.61 | 0.69 |
| letter.33 | 0.46 | | 0.30 | | | 0.31 | 0.69 | 0.70 |
| letter.34 | 0.54 | | 0.38 | | | 0.43 | 0.57 | 0.66 |
| letter.58 | 0.47 | | 0.20 | | | 0.28 | 0.72 | 0.78 |
| matrix.45 | 0.40 | | | | 0.66 | 0.59 | 0.41 | 0.27 |
| matrix.46 | 0.40 | | | | 0.26 | 0.24 | 0.76 | 0.65 |
| matrix.47 | 0.42 | | | | | 0.23 | 0.77 | 0.79 |
| matrix.55 | 0.28 | | | | | 0.12 | 0.88 | 0.65 |
| rotate.3 | 0.36 | 0.61 | | | | 0.50 | 0.50 | 0.26 |
| rotate.4 | 0.41 | 0.61 | | | | 0.54 | 0.46 | 0.31 |
| rotate.6 | 0.40 | 0.49 | | | | 0.41 | 0.59 | 0.39 |
| rotate.8 | 0.32 | 0.53 | | | | 0.40 | 0.60 | 0.26 |

ω continued

With eigenvalues of:

| | | | | |
|------|------|------|------|------|
| g | F1* | F2* | F3* | F4* |
| 3.04 | 1.32 | 0.46 | 0.42 | 0.55 |

general/max 2.3 max/min = 3.17
 mean percent general = 0.58 with sd = 0.2 and cv of 0.35
 Explained Common Variance of the general factor = 0.53

The degrees of freedom are 62 and the fit is 0.05
 The number of observations was 1525 with Chi Square = 70.19 with prob < 0.22
 The root mean square of the residuals is 0.01
 The df corrected root mean square of the residuals is 0.02
 RMSEA index = 0.009 and the 90 % confidence intervals are 0 0.014
 BIC = -384.25

Compare this with the adequacy of just a general factor and no group factors
 The degrees of freedom for just the general factor are 104 and the fit is 0.78
 The number of observations was 1525 with Chi Square = 1186.18 with prob < 5e-183
 The root mean square of the residuals is 0.09
 The df corrected root mean square of the residuals is 0.09
 RMSEA index = 0.083 and the 90 % confidence intervals are 0.078 0.085
 BIC = 423.88

Measures of factor score adequacy

| | g | F1* | F2* | F3* | F4* |
|---|------|------|-------|-------|------|
| Correlation of scores with factors | 0.83 | 0.80 | 0.53 | 0.56 | 0.71 |
| Multiple R square of scores with factors | 0.69 | 0.64 | 0.28 | 0.32 | 0.50 |
| Minimum correlation of factor score estimates | 0.37 | 0.28 | -0.44 | -0.37 | 0.00 |

Total, General and Subset omega for each subset

| | g | F1* | F2* | F3* | F4* |
|--|------|------|------|------|------|
| Omega total for total scores and subscales | 0.86 | 0.77 | 0.69 | 0.64 | 0.53 |

Data set from Preacher and Hayes

R code

```
# from Preacher and Hayes (2004)
sobel <- structure(list(SATIS = c(-0.59, 1.3, 0.02, 0.01, 0.79, -0.35,
-0.03, 1.75, -0.8, -1.2, -1.27, 0.7, -1.59, 0.68, -0.39, 1.33,
...
"Therapy", "Attributional Positivity"), .Names = c("SATIS", "THERAPY",
"ATTRIB"))
#n.iter set to 50 (instead of default of 5000) for speed of example
mediate(1,2,3,sobel,n.iter=50) #The example in Preacher and Hayes
```

The DV (Y) was SATIS . The IV (X) was THERAPY . The mediating variable(s) = ATTRIB .

Total Direct effect(c) of THERAPY on SATIS = 0.76 S.E. = 0.31 t direct = 2.5
 Direct effect (c') of THERAPY on SATIS removing ATTRIB = 0.43 S.E. = 0.32 t di
 Indirect effect (ab) of THERAPY on SATIS through ATTRIB = 0.33
 Mean bootstrapped indirect effect = 0.31 with standard error = 0.16 Lower CI = 0.07
 R2 of model = 0.31

To see the longer output, specify short = FALSE in the print statement

Full output

Total effect estimates (c)

| | SATIS | se | t | Prob |
|---------|-------|------|-----|--------|
| THERAPY | 0.76 | 0.31 | 2.5 | 0.0186 |

Direct effect estimates (c')

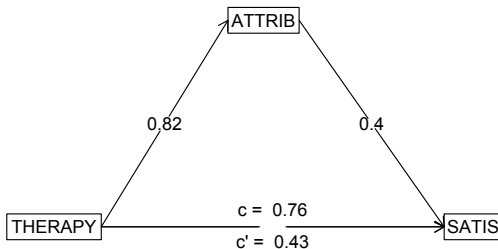
| | SATIS | se | t | Prob |
|---------|-------|------|------|-------|
| THERAPY | 0.43 | 0.32 | 1.35 | 0.190 |
| ATTRIB | 0.40 | 0.18 | 2.23 | 0.034 |

'a' effect estimates

| | THERAPY | se | t | Prob |
|--------|---------|-----|------|--------|
| ATTRIB | 0.82 | 0.3 | 2.74 | 0.0106 |

The Preacher mediation example

Mediation model



Take the data example from Hayes

R code

```
C.pmi <- structure(c(0.251232840197254, 0.119718779155005, 0.157470345195255,
0.124533519925363, 0.03052112488338, 0.0734039717446355, 0.119718779155005,
...
33.6509729441557), .Dim = c(6L, 6L), .Dimnames = list(c("cond",
"pmi", "import", "reaction", "gender", "age"), c("cond", "pmi",
"import", "reaction", "gender", "age")))

#n.iter set to 50 (instead of default of 5000) for speed of example
mediate(y="reaction", x = "cond", m=c("pmi", "import"), data=C.pmi, n.obs=123, n.iter=50)
```

```
Call: mediate(y = "reaction", x = "cond", m = c("pmi", "import"), data = C.pmi,
n.obs = 123, n.iter = 50)
```

The DV (Y) was reaction . The IV (X) was cond . The mediating variable(s) = pmi import

```
Total Direct effect(c) of cond on reaction = 0.5 S.E. = 0.28 t direct = 1.79
Direct effect (c') of cond on reaction removing pmi import = 0.1 S.E. = 0.24 t
Indirect effect (ab) of cond on reaction through pmi import = 0.39
Mean bootstrapped indirect effect = 0.7 with standard error = 0.17 Lower CI = 0.39
R2 of model = 0.33
```

To see the longer output, specify short = FALSE in the print statement

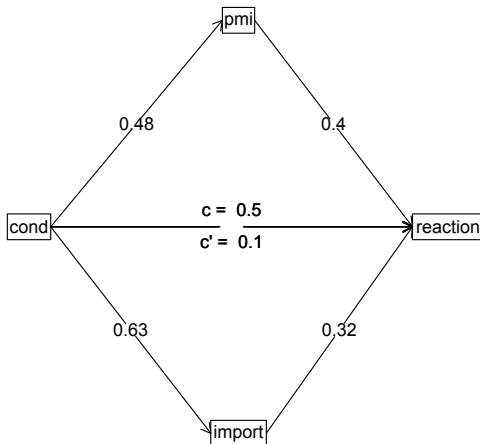
Full output

```
Total effect estimates (c)
  reaction se t Prob
cond      0.5 0.28 1.79 0.0766
```

```
Direct effect estimates (c')
  reaction se t Prob
cond      0.1 0.24 0.43 0.66601
```

The Hayes example mediation

Mediation model



The “New” Psychometrics

1. Classical Test theory examines responses assuming items are equivalent, or at least congeneric equivalent
2. Item Response Theory models item difficulty as well as item discrimination
3. Although seemingly very different models, factor analysis of categorical items (using tetrachoric or polychoric correlations is equivalent to IRT 2 PL models.
4. Rasch model is just a 1 PL model where items differ in difficulty, but not discrimination.
5. 2PL has difficulty and discrimination estimated from factor analysis of tetrachoric/polychoric items.

R code

```
f1 <- irt.fa(ability)
```

```
f1 <- irt.fa(ability)
```

```
> f1
```

```
Item Response Analysis using Factor Analysis
```

```
Call: irt.fa(x = ability)
```

```
Item Response Analysis using Factor Analysis
```

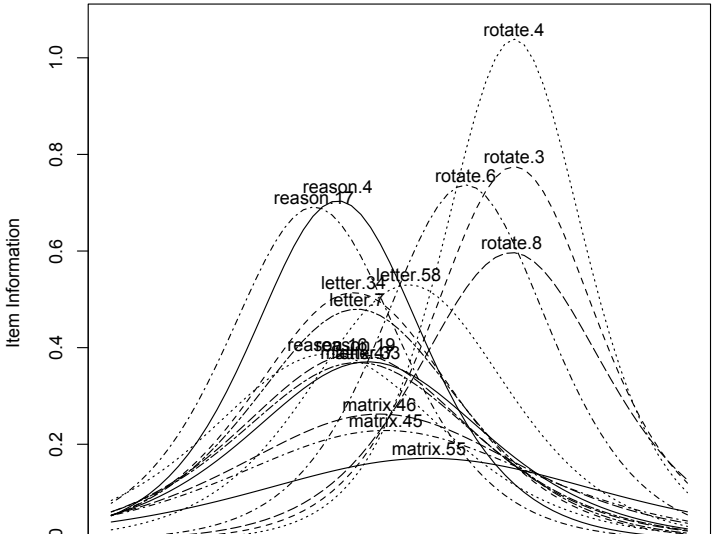
```
Summary information by factor and item
```

```
Factor = 1
```

| | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|-----------|------|------|------|------|------|------|------|
| reason.4 | 0.05 | 0.24 | 0.64 | 0.53 | 0.16 | 0.03 | 0.01 |
| reason.16 | 0.08 | 0.22 | 0.38 | 0.31 | 0.14 | 0.05 | 0.01 |
| reason.17 | 0.08 | 0.33 | 0.69 | 0.42 | 0.11 | 0.02 | 0.00 |
| reason.19 | 0.06 | 0.17 | 0.35 | 0.36 | 0.19 | 0.07 | 0.02 |
| letter.7 | 0.05 | 0.18 | 0.41 | 0.44 | 0.20 | 0.06 | 0.02 |
| letter.33 | 0.05 | 0.15 | 0.31 | 0.36 | 0.20 | 0.08 | 0.02 |
| letter.34 | 0.05 | 0.19 | 0.45 | 0.46 | 0.20 | 0.06 | 0.01 |
| letter.58 | 0.02 | 0.09 | 0.30 | 0.53 | 0.35 | 0.12 | 0.03 |
| matrix.45 | 0.05 | 0.11 | 0.19 | 0.23 | 0.17 | 0.09 | 0.04 |
| ... | | | | | | | |
| Test Info | 0.67 | 2.11 | 4.73 | 5.83 | 5.28 | 2.55 | 0.69 |
| SEM | 1.22 | 0.69 | 0.46 | 0.41 | 0.44 | 0.63 | 1.20 |

FA solution with tetrachoric correlations

Item information from factor analysis



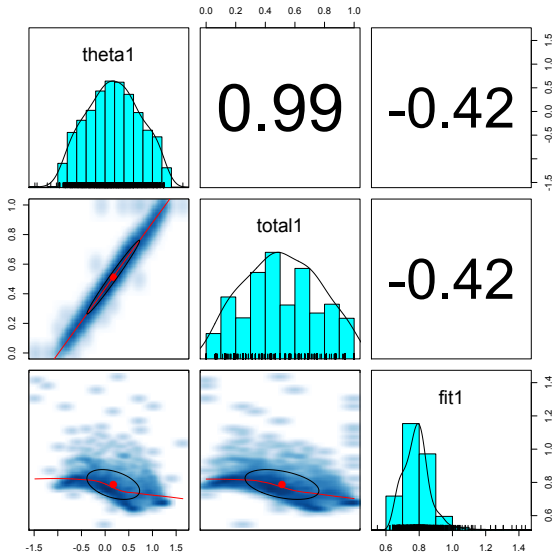
IRT based scoring and Classical Test Theory based scoring

1. CTT and IRT based scores correlate almost perfectly without missing data
2. With lots of missing data, and different items for different people, IRT based scores provide more subtle distinctions.
3. `scoreIrt.2p1` and `scoreIrt.1p1` will do IRT based scores.
4. By default, will find the irt based parameters and then do the scoring.

R code

```
ability.irt <- irt.fa(ability)
ability.scores <- scoreIrt(ability.irt,ability)
pairs.panels(ability.scores, smoother=TRUE)
```

CTT and IRT based scores are almost identical



Multilevel reliability

1. Classic reliability measures assess the variance of between person differences compared to error of the measurement.

$$\rho_{xx} = \frac{1 - \sigma_e^2}{\sigma_x^2} \quad (1)$$

2. Multilevel reliability is a series of generalizability coefficients, generalizing over items, over time, time x items,

$$R_{kF} = \frac{\sigma_{id}^2 + (\sigma_{idxitems}^2/m)}{\sigma_{id}^2 + (\sigma_{idxitems}^2/m) + \sigma_{error}^2/(km)} \quad (2)$$

From Equation 6 (Shrout & Lane, 2012, p 310). See Shrout & Lane (2012) for five other generalizability formula.

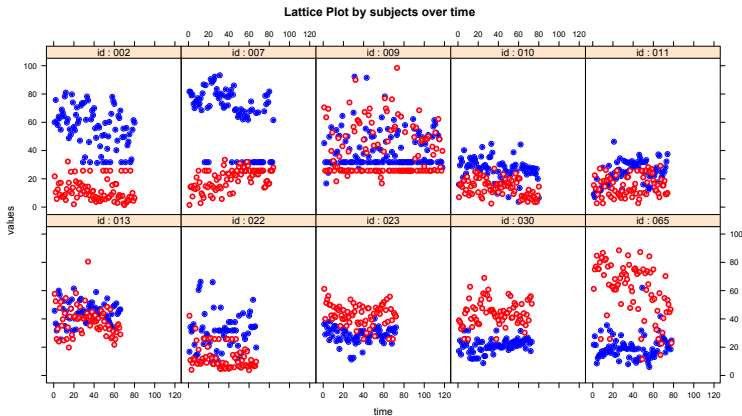
3. Implemented in *psych* as `mlr` or `multilevel.reliability`
4. Also simulations using `sim.multi`
5. I show the data from Fisher (2015) who reports 10 subjects measured over 60 (or more) days on 28 affect items.
6. (Download the R data files, minor rearrangement and reliability measurements)

Table: The `multilevel.reliability` function estimates of the generalizability coefficients for the positively and negatively valenced items from Fisher (2015). RkF is the reliability of average of all ratings across all items and times (Fixed time effects), R1R is the generalizability of a single time point across all items (Random time effects), RkR is the generalizability of average time points across all items (Random time effects), Rc is the generalizability of change (fixed time points, fixed items), RkRn is the generalizability of between person differences averaged over time (time nested within people) and Rcn is the generalizability of within person variations averaged over items (time nested within people).

| Multilevel reliability estimates | | |
|----------------------------------|----------------|----------------|
| Coefficient | Positive items | Negative items |
| RkF | 1.00 | 1.00 |
| R1R | 0.80 | 0.77 |
| RkR | 1.00 | 1.00 |
| Rc | 0.72 | 0.71 |
| RkRn | 1.00 | 1.00 |
| Rcn | 0.64 | 0.59 |



Assessing reliability of within subject differences in affect. Data from Fisher (2015)



psych includes some very old ideas

1. Schmid-Leiman (Schmid & Leiman, 1957) transformations from correlated factor structures to higher order structures.
2. Dwyer extension (Dwyer, 1937; Mosier, 1938; Horn, 1973) to extend a factor solution to more variables.
3. This can be used to extend other variables into a factor space, or to relate two domains to each other.

Extend a data set into another

First, create the data set

R code

```
set.seed(42)
d <- sim.item(12)      #two orthogonal factors
R <- cor(d)
Ro <- R[c(1,2,4,5,7,8,10,11),c(1,2,4,5,7,8,10,11)]
Roe <- R[c(1,2,4,5,7,8,10,11),c(3,6,9,12)]
fo <- fa(Ro,2)
fe <- fa.extension(Roe,fo)
fa.diagram(fo,fe=fe)
```

fe

Call: fa.extension(Roe = Roe, fo = fo)

Standardized loadings (pattern matrix) based upon correlation matrix

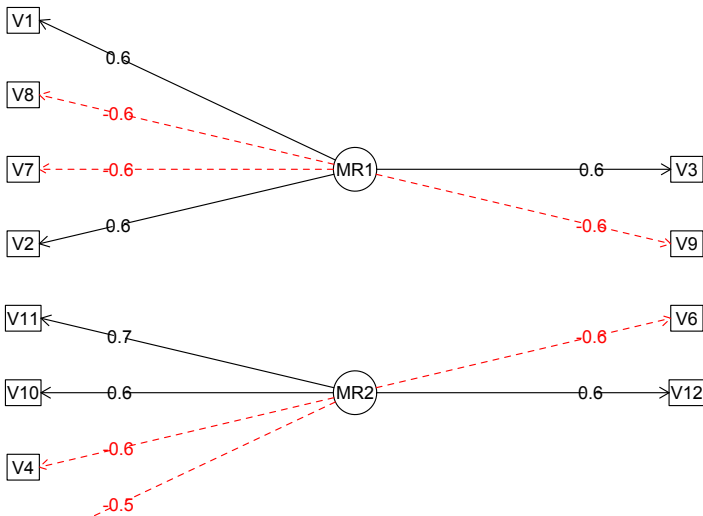
| | MR1 | MR2 | h2 | u2 |
|-----|-------|-------|------|------|
| V3 | 0.63 | -0.02 | 0.39 | 0.61 |
| V6 | 0.04 | -0.61 | 0.37 | 0.63 |
| V9 | -0.61 | 0.01 | 0.38 | 0.62 |
| V12 | -0.06 | 0.58 | 0.33 | 0.67 |

| | MR1 | MR2 |
|-----------------------|------|------|
| SS loadings | 0.77 | 0.69 |
| Proportion Var | 0.19 | 0.17 |
| Cumulative Var | 0.19 | 0.37 |
| Proportion Explained | 0.53 | 0.47 |
| Cumulative Proportion | 0.53 | 1.00 |

| | MR1 | MR2 |
|-----|------|------|
| MR1 | 1.00 | 0.00 |

Factor extension

Factor analysis and extension



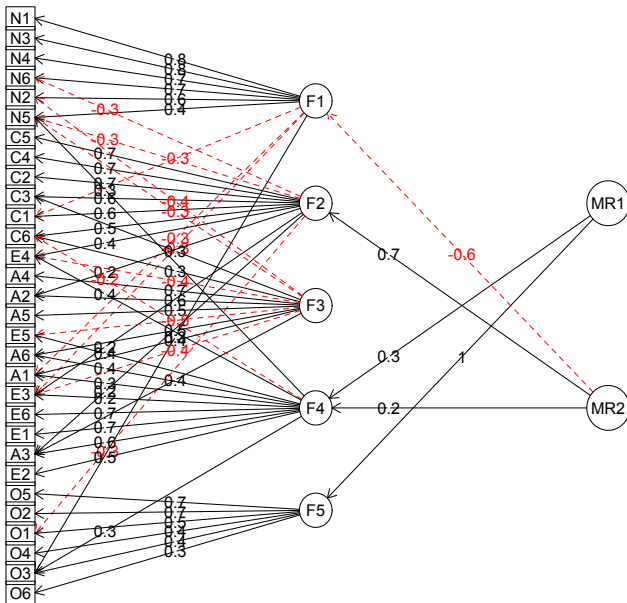
Hierarchical factor analysis

R code

```
neo52 <- fa.multi(neo, 5, 2)  
fa.multi.diagram(neo52)
```



Hierarchical (multilevel) Structure



ESEM can be thought of as factor extension from A to B and B to A

1. If we have two sets of variables that show factor structures within each set
2. And then link the factor structures.
3. Tjhis can be done in SEM, but here show how to do exploratory SEM
4. We make up a toy data set

R code

```
#make up a sem like problem using sim.structure
fx <-matrix(c( .9, .8, .6, rep(0,4) , .6, .8, -.7), ncol=2)
fy <- matrix(c(.6, .5, .4), ncol=1)
rownames(fx) <- c("V", "Q", "A", "nach", "Anx")
rownames(fy) <- c("gpa", "Pre", "MA")
Phi <-matrix( c(1,0, .7, .0, 1, .7, .7, .7, 1), ncol=3)
gre.gpa <- sim.structural(fx, Phi, fy)
print(gre.gpa)
```

Call: sim.structural(fx = fx, Phi = Phi, fy = fy)

\$model (Population correlation matrix)

| | V | Q | A | nach | Anx | gpa | Pre | MA |
|------|------|------|-------|-------|-------|-------|-------|-------|
| V | 1.00 | 0.72 | 0.54 | 0.00 | 0.00 | 0.38 | 0.32 | 0.25 |
| Q | 0.72 | 1.00 | 0.48 | 0.00 | 0.00 | 0.34 | 0.28 | 0.22 |
| A | 0.54 | 0.48 | 1.00 | 0.48 | -0.42 | 0.50 | 0.42 | 0.34 |
| nach | 0.00 | 0.00 | 0.48 | 1.00 | -0.56 | 0.34 | 0.28 | 0.22 |
| Anx | 0.00 | 0.00 | -0.42 | -0.56 | 1.00 | -0.29 | -0.24 | -0.20 |
| gpa | 0.38 | 0.34 | 0.50 | 0.34 | -0.29 | 1.00 | 0.30 | 0.24 |
| Pre | 0.32 | 0.28 | 0.42 | 0.28 | -0.24 | 0.30 | 1.00 | 0.20 |
| MA | 0.25 | 0.22 | 0.34 | 0.22 | -0.20 | 0.24 | 0.20 | 1.00 |

\$reliability (population reliability)

| | V | Q | A | nach | Anx | gpa | Pre | MA |
|--|------|------|------|------|------|------|------|------|
| | 0.81 | 0.64 | 0.72 | 0.64 | 0.49 | 0.36 | 0.25 | 0.16 |

○○
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Exploratory Structural Equation Modling

R code

```
example <- esem(gre.gpa$model, varsX=1:5, varsY=6:8, nfX=2, nfY=1,
n.obs=1000, plot=FALSE)
```

```
> example
```

```
Exploratory Structural Equation Modeling Analysis using method = minres
```

```
Call: esem(r = gre.gpa$model, varsX = 1:5, varsY = 6:8, nfX = 2, nfY = 1,
```

```
n.obs = 1000, plot = FALSE)
```

```
For the 'X' set:
```

```
      MR1  MR2
V      0.91 -0.06
Q      0.81 -0.05
A      0.53  0.57
nach -0.10  0.81
Anx   0.08 -0.71
```

```
For the 'Y' set:
```

```
      MR1
gpa  0.6
Pre  0.5
MA   0.4
```

```
Correlations between the X and Y sets.
```

```
      X1  X2  Y1
X1  1.00 0.19 0.68
X2  0.19 1.00 0.67
Y1  0.68 0.67 1.00
```

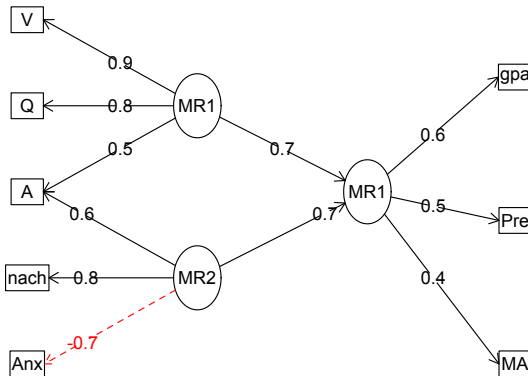
```
The degrees of freedom for the null model are 56 and the empirical chi square function
```

```
The degrees of freedom for the model are 7 and the empirical chi square function was 21
```

```
with prob < 0.0027
```

ESEM of our toy problem

Exploratory Structural Model



UseR! and the *psych*

1. Not shown

- Basic IRT analysis using `irt.fa`
- Statistics for each group using `describeBy`, `statsBy` and `faBy`
- Circular statistics for diurnal mood data using `cosinor`, `circadian.mean`, etc.
- Simulation of multiple types of data structures using `sim`, `sim.items`, ...
- Convenient \LaTeX output functions

2. Tutorials for R and the *psych* package at

<http://personality-project.org/r>

- Guide to *psych* package
<http://personality-project.org/r/psych>
- Vignette for *psych* package at
<http://personality-project.org/r/psych/vignettes/overview.pdf>

3. Slides from today at: <http://personality-project.org/r/tutorials/swpa/swpa.17.pdf>

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